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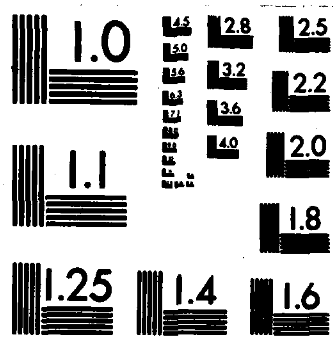

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INCORPORATING CONTROL INTO THE OPTIMAL STRUCTURAL DESIGN OF LARGE FLEXIBLE SPACE STRUCTURES

THESIS

Thomas V. Muckenthaler
Capt USAF

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Thomas V. Muckenthaler
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Thomas V. Muckenthaler

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ABSTRACT

An eigenspace optimization approach is used to incorporate optimal control into the structural design process for large flexible space structures. The equations of motion for an uncontrolled system are developed by deriving the kinetic and potential energy for the system and then using assumed modes to discretize the energies. These expressions are then linearized, the Lagrangian formed, and Lagrange's equations written for the system. An existing optimal control law is incorporated to form the equations of motion for the controlled system. A parameter optimization technique is used to minimize the mass of the Draper/RPL Configuration model involving eigenspace optimization. A computer algorithm is developed that effectively optimizes a global structural parameter vector to minimize the mass of the model, while constraining specified eigenvalues. The eigenvalue sensitivities are passed to a constrained function minimization program called CONMIN which minimizes the mass of the appendages. The constraints imposed restrict the first eigenvalue to the left half plane and the natural frequency of the third eigenvalue to a specified stable region. The result is an algorithm that incorporates an existing optimal control law into the structural optimization process.

INCORPORATING CONTROL INTO THE OPTIMAL STRUCTURAL DESIGN OF LARGE FLEXIBLE SPACE STRUCTURES

I. INTRODUCTION

President Reagan has directed NASA to begin work on a permanent manned space station. The Russians just completed a record stay in space and the space shuttle will begin monthly flights very soon. The demand for large, controlled flexible space structures will soon increase greatly. These structures must not only be stable, but must also be designed to minimize their mass. Even with the advent of the space shuttle, transporting massive objects into Earth orbit is costly. Efforts to minimize the mass of the large space structure usually result in a highly flexible structure that can pose a difficult control problem.

Current methods of structural design generally do not incorporate control into the optimization process. The result is a structure that is optimally designed with respect to strength or vibration or some other performance parameter and then an optimum control system is designed for the existing structure. Therefore, optimal design from a structural standpoint can represent varying structural parameters to minimize the mass, whereas for a control designer, optimization means controlling the number and size of controllers such that a specified performance index is minimized [9].

While many recent papers have addressed the problem of optimizing a control system for a given space structure, this paper will examine methods of designing a large flexible space structure with a given control system in mind.

A very stiff , and hence a more massive, large space structure will require less control energy than a like structure that has much less mass and is more flexible. Also the greater it's mass, the more expensive it is to transport into orbit. Therefore, there is a tradeoff between minimization of mass of a structure and the ability to control it. The minimum mass for a structure is described by Hale in [1] to be the essential mass to accomplish the mission for which the structure is designed. He goes on to say that this structure can be a collection of discrete masses or be spatially distributed. When using discrete actuators on a spatially distributed system the amount of control energy expended can be reduced by stiffening the structure. This effectively distributes the control forces over the 'essential structure', but it is at the expense of increasing the total mass of the structure.[1] Any variation in the parameters must retain controlability and stability of the entire structure.

HISTORY OF RELATED EFFORTS

Hale and Lisowski in [1] develop an analytical approach to the optimal structural design/control design problem. They use linear quadratic-cost control theory to design an optimal control for known structural parameters. A weighted total mass term is added to the cost functional and the structural parameters are varied to find the minimum total cost. Discrete equations are the result of spatially discretizing the partial differential equations that represent the complex structure composed of many distributed components. The calculus of variations is used to derive the necessary conditions for an optimum of the problem. The result is a hybrid system of coupled nonlinear equations with 'no hope for an analytical solution'. A numerical solution composed of two parts is discussed. The two parts are: 1) find a control design that is optimal for given structural parameters, and 2) a procedure to update the structural parameters. Thus the procedure is an iterative one and because the cost functional is linear in the structural parameters, the second partial derivatives of the functional with respect to the structural parameters are zero. As stated in the report, when using this approach, care must be exercised as the extremal solution may only be a relative minima or maxima. One important aspect of this work is that nonlinear effects are not included.

In [2] Hale and Lisowski use the same approach as in

[1] for a reduced order problem. The transformation is accomplished using the force-free eigenvectors to form a new basis for the modal coordinates. A new set of equations are formed to approximate the solution to the original problem. How well this approximate solution represents the actual solution depends on the participation of the residual modes. Control derivative penalties are used to reduce the level of excitation of these modes. Only open loop maneuvers between specified states in a specified time interval were considered and the results indicate that the problem can be made more practical by using a reduced order structural model.

In [9] Venkayya and Tischler report on the effects of modification of structural parameters on the dynamic response of flexible structures with and without active controls. The control problem and structural optimization are conducted independently with the structural optimization based on the minimization of the mass with the fundamental frequency as a constraint. The procedure used addresses the optimal control design with a quadratic performance index involving the energy of the vibrating structure and the actuators. External disturbances which initiate or continue the vibration of the system are not considered. A state space representation for the open loop system with modal reduction is used to derive an expression for the damping ratio and frequencies. To determine the optimum feedback control, a quadratic performance index with specified weighting matrices

is minimized. This is accomplished through the solution of the steady state matrix Riccati equation.

The algorithm used for the structural optimization is developed in detail in [6]. The objective is to minimize the mass of a vibrating structure subject to constraining the fundamental frequency to a specified value. A scaling factor is used such that the frequency of the scaled design is equal to the specified fundamental frequency of the unscaled equation. The scaling factor is iterated from 0 to 1, with the final value converging to the desired result. The paper shows that modification of the structural parameters did not significantly alter the control design with no external disturbances, but the changes are expected by the authors to have a more pronounced effect when an external disturbance is present.

Khot, Venkayya, and Eastep in [3], examine the dynamic response of large space structures due to changes in the stiffness coefficients. The paper uses a procedure to examine the effect of structural modifications on the pointing error of a tetrahedral truss. Essentially, the cross-sectional areas of the truss members are optimized to mimimize a displacement function and in a separate algorithm are used to minimize the mass of the structure, while constraining the frequency distribution. An iteration process is used until the optimality criterion are satisfied. They show that by changing the stiffness of the members based on the optimum

design criterion, an improvement in the dynamic response of the structure to a external disturbance can be made. The improvement depends on the criteria used and the initial conditions.

In [4], Junkins, Bodden, and Kamat develop an eigenspace optimization approach for design of feedback controllers for maneuvers/vibration arrests of flexible structures. They state that most structural and control optimality criteria can be stated explicitly or implicitly in terms of the eigenvalues, eigenvectors, or directly as a function of a global structural and control parameter vector. The eigenvalues and eigenvectors are shown to be a function of the structural and control parameters and an algorithm is developed to compute their partial derivatives. A differential equation model for the Draper/RPL Configuration is used to demonstrate the approach. In the numerical example presented, the structural parameters, sensor locations, and actuator locations are fixed and the control gains which stabilize the first few modes with the smallest sum square are solved for. A performance measure and constraints are defined in terms of the eigensolution with explicit dependence upon the control parameters. The algorithm is applied to place the system's closed loop eigenvalues in a specified region and subject to this condition minimize a robustness measure, the sensitivity of the eigenvalues.

APPROACH USED

Following the work of Junkins, Bodden, and Kamat in [4] an eigenspace optimization approach is used to incorporate optimal control into the structural design process of large flexible space structures. An expression for the rates of change of eigenvalues and eigenvectors with respect to parameters of the system is required. Wittrick in [9] determined the first derivatives of the eigenvalues for a self-adjoint system and Fox and Kapoor in [10] developed expressions for the first derivatives ofthe eigenvectors. Plaut and Huseyin in [7] derived general expressions for the derivatives of eigenvalues and eigenvectors in non-self-adjoint systems. Junkins, et al, used this work to develop an algorithm to minimize an optimality criteria and satisfy constraints stated as a function of the parameters of the system (p) and the eigenvalues. As stated in their paper a necessity for the optimization algorithm is to efficiently compute the partial derivatives of the eigenvalues and eigenvectors with respect to the structural and control design parameters (p).

After developing their algorithm, they applied it to a mathematical model of the Draper/RPL Configuration and solved for the optimum gain matrices holding all the structural parameters, sensor locations, and actuator locations constant. These gains are those which provide specified values for the first few modal frequencies and

damping ratios with the smallest sum square of the elements of the gain matrices. This represents a sub-optimization as other possible optimization parameters were fixed at nominal values. This paper will continue their work and seek to optimze this same model in the sense of minimizing the mass.

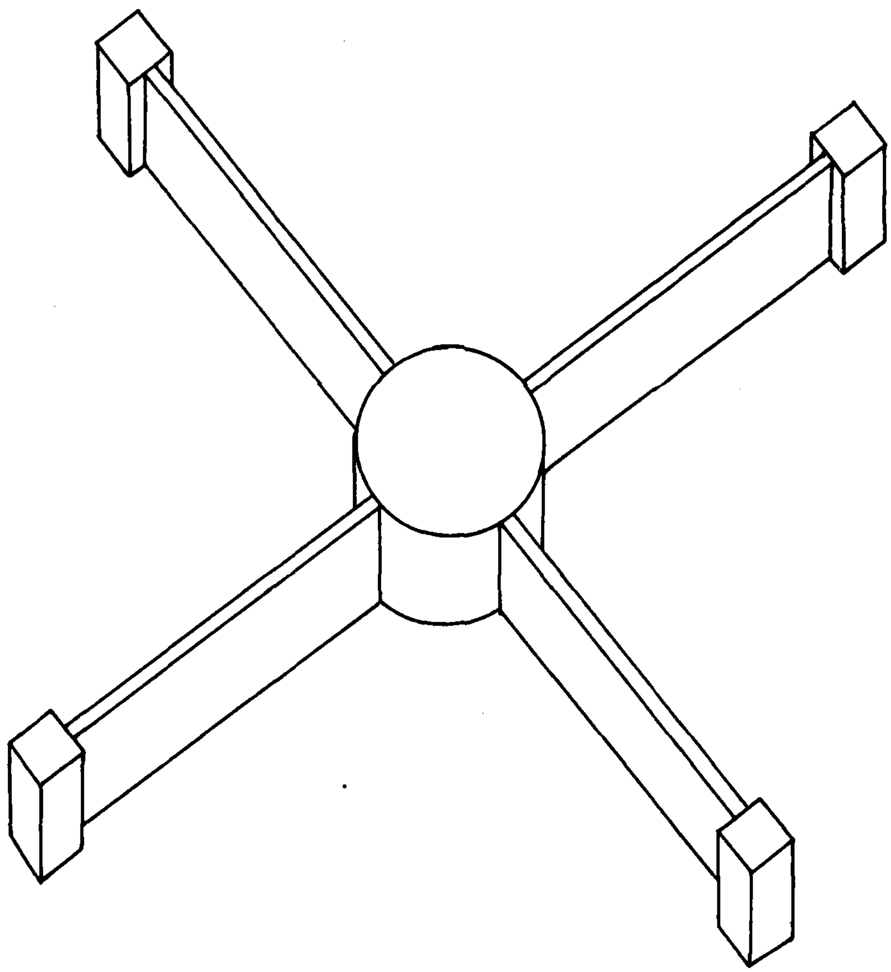

Figure 1. Draper/RPL Configuration Model

II. DEVELOPMENT OF MATHEMATICAL MODEL

In order to develop an analytical model it is necessary to develop an idealized model of the structural system to be studied. The structure used as a model for this report is the 'Draper / RPL Configuration' model illustrated in Figure 1. The structure consists of a rigid central hub with four structurally identical flexible appendages cantilevered radially from the hub. The central hub is assumed to be supported on an air bearing table. Experimental work concerning the planar rotational/ vibrational dynamics of this U.S.A.F. Rocket Propulsion Laboratory demonstration model is presently being conducted at the Charles Stark Draper Laboratory.[4]

The cantilevered appendages modeled here can be thought of as a continuous beam with a lumped mass at the end. The differential equations of motion can then be thought of as the mathematical language which describes the analytical model. This set of differential equations will be referred to as the mathematical model of the structure. The mathematical model development begins with the derivation of the equations of motion for an unforced system. Expressions for the kinetic and potential energy are developed for the system. These are discretized using the assumed modes method and terms of order higher than two ignored. The mass and stiffness matrices are then identified. Following the

unforced or uncontrolled model is the derivation of the controlled system. The control gains used will be those found optimum by Junkins in [4]. The next step in the analysis is to solve the differential equations to obtain the dynamical response of the free vibration and also the response of the controlled system.

DEFINITION OF TERMS FOR EQUATIONS OF MOTION

$\bar{r}_i$ is a position vector of the ith appendage

T is the total kinetic energy of the system

T_{bo} is the kinetic energy of the rigid hub

T_{bi} is the kinetic energy of the ith appendage

$\hat{i}$, $\hat{j}$, $\hat{k}$ represents the body fixed reference frame

u, v are the deflections of the appendages in the $\hat{i}$, $\hat{j}$ directions, respectively.

$\bar{w}$ is the angular rotation of the body fixed reference frame with respect to the inertial frame.

R is the radius of the hub

L is the length of the appendages

t is the thickness of the appendages

h is the height of the appendages

I_{2c} is the rotational moment of inertia of the tip mass.

m_o is the mass of the hub

m_2 is the mass of the point masses

ASSUMPTIONS

The longitudinal and out of plane vibration of the appendages are of much higher frequency than the transverse vibrations and are neglected. Anti-symmetric deformations were assumed such that the deflection of the first appendage was equal in magnitude but opposite in direction to the second appendage and the deflection of the third appendage was equal and opposite to the fourth, see figure 2. Also since the hub is supported by an air bearing table, the force of gravity was neglected in the derivation of the potential energy. The damping coefficient was assumed to be equal to zero. All four appendages are assumed to be structurally identical, that is

$$L_1 = L_2 = L_3 = L_4 = L$$

$$t_1 = t_2 = t_3 = t_4 = t$$

$$h_1 = h_2 = h_3 = h_4 = h$$

and all four tip masses are identical and equal to m_2.

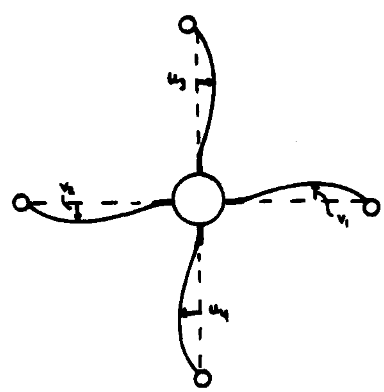

Figure 2. Anti-Symmetric Deflections

EQUATIONS OF MOTION FOR THE FREE SYSTEM

The equations of motion for the uncontrolled system or free vibrations are derived by finding the kinetic and potential energy for the system and then using assumed modes to descritize the energies. These were then reduced by truncating terms of order higher than two, the Lagrangian formed, and Lagrange's Equations written for the system. In this derivation an expression for the kinetic energy is developed first, followed by the development of the potential energy. Figure 3 depicts the Draper / RPL model with the appendages numbered and the body fixed $\hat{i}$, $\hat{j}$, $\hat{k}$ reference frame shown.

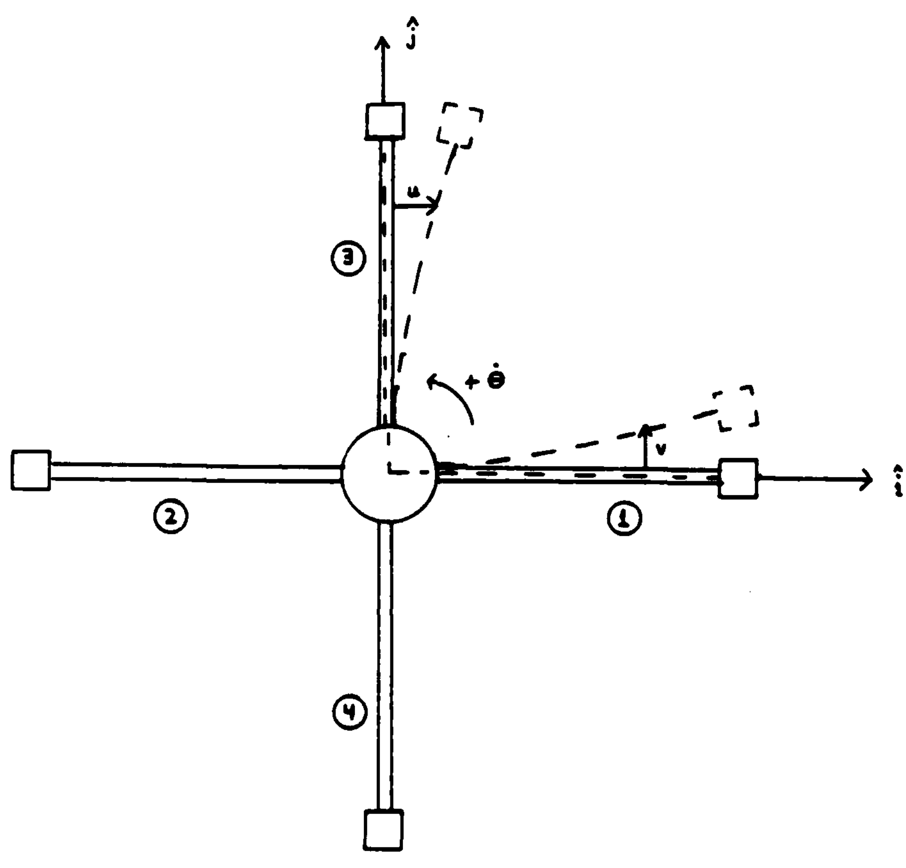

Figure 3. Model with $\hat{i}$, $\hat{j}$, $\hat{k}$ Reference Frame.

The kinetic energy of the hub alone is just that of a solid cylinder and is written as

$$T_{bo} = 1/2 \; I_o \; w^2 \tag{1}$$

where $\quad I_o = 1/2 \; m_o \; R^2$

To determine the kinetic energy of the first appendage a position vector is written as

$$\bar{r}_1 = x \hat{i} + v \hat{j}$$

The time rate of change of the elements of appendage 1 are determined from the first derivative with respect to an inertial reference frame. All derivations here are with respect to an inertial reference frame.

$$d \bar{r}_1 / dt = \dot{v} \hat{j} + \bar{w} \times \bar{r}_1$$

$$\dot{\bar{r}}_1 = - v w \hat{i} + (\dot{v} + x w) \hat{j} \tag{2}$$

The kinetic energy of the first appendage is the inner product or dot product of $\dot{\bar{r}}_1 \cdot \dot{\bar{r}}_1$ integrated over the mass from the end of the hub to the end of the appendage.

$$\dot{\bar{r}}_1 \cdot \dot{\bar{r}}_1 = (v^2 w^2 + \dot{v}^2 + 2 \dot{v} x w + x^2 w^2) \tag{3}$$

$$T_{b1} = 1/2 \int_{x=R}^{x=R+L} \dot{\bar{r}}_1 \cdot \dot{\bar{r}}_1 \, dm$$

Let $I_1 = \displaystyle\int_{x=R}^{x=R+L} x^2 \, dm$

Then the kinetic energy for the first appendage can be written as

$$T_{b1} = 1/2\ I_1\ w^2 + 1/2 \int_{R}^{R+L} (v^2 w^2 + \dot{v}^2 + 2\ \dot{v}\ w\ x)\ dm \qquad (4)$$

The kinetic energy of the tip mass is found by writing the position vector , taking the appropriate derivatives and computing the dot product. Therefore :

$$\bar{r}_2 = (R + L)\ \hat{i} + v(\ x=R+L\ )\ \hat{j} \qquad (5)$$

where v is evaluated at x = R+L . Taking the first derivative yields :

$$\dot{\bar{r}}_2 = \dot{v}\ \hat{j} + \bar{w} \times \bar{r}_2$$

$$= (\dot{v} + w\ (R+L)\ )\ \hat{j}\ -\ v\ w\ \hat{i} \qquad (6)$$

$$\dot{r}_2\ .\ \dot{r}_2 = V_2^{\ 2}$$

$$= \dot{v}^2 + 2\ \dot{v}\ w\ (R+L) + w^2 (R+L)^2 + v^2\ w^2$$

The angular velocity of the tip mass is the angular velocity of the body fixed reference frame plus a component due to the deflection of the end of the appendage where the tip mass is located.

$$w_2 = w + \dot{v}\ '$$

where

$$\dot{v}\ ' = d\ (\ \partial v\ /\ \partial x\ )\ /\ dt \quad \text{evaluated at } x = R+L \qquad (7)$$

The kinetic energy of the tip mass can now be expressed as

$$T_{b2} = 1/2\ m_2\ V_2^{\ 2} + I_{2c}\ \ w_2^{\ 2} \qquad (8)$$

While the rotational moment of inertia of a tip mass is small, it will be included in the general formulation.

In a similar manner, the kinetic energy of the third appendage and its tip mass were derived and are shown here as

$$T_{b3} = 1/2\ I_3\ w^2 + 1/2 \int_{y=R}^{y=R+L} (u^2 w^2 + \dot{u}^2 + 2\,\dot{u}\ w\ y)\ dm \qquad (9)$$

where

$$I_3 = \int_{y=R}^{y=R+L} y^2 \; dm$$

and the expression for the tip mass of the third appendage is

$$T_{b4} = 1/2 \; m_2 \; V_4{}^2 \; + \; I_{2c} \; w_4{}^2$$

where

$$V_4{}^2 = \dot{u}^2 + 2 \; \dot{u} \; w \; (R+L) \; + \; u^2 \; w^2 \; + \; w^2(R+L)^2 \qquad (10)$$

and $\quad w_4 = w + \dot{u}'$

where $\quad \dot{u}' = d\,(\,\partial u / \partial y\,)\,/\,dt \quad$ evaluated at $y = R+L$

Now, since only the case of antisymmetric deformation is being considered , where

$$u_3 = -u_4 \qquad , \qquad v_1 = -v_2$$

it follows that the kinetic energy of the first appendage and tip mass is equal in magnitude to the kinetic energy of the second appendage and its tip mass. Likewise the kinetic energy of the third appendage and tip mass is equal in magnitude to the kinetic energy of the fourth appendage and point mass. Therefore, the total kinetic energy of the

system can be written as the sum of the individual terms as:

$$T = T_{bo} + 2\ T_{b1} + 2\ T_{b2} + 2\ T_{b3} + 2\ T_{b4} \tag{11}$$

To expand this expression let :

$$I_c = 1/2\ I_o + 2\ I_1 + 2\ m_2\ (R+L)^2$$

The total kinetic energy can be regrouped as :

$$T = (I_c + 2\ I_{2c}) w^2 + \rho t\ h \int (u^2 w^2 + \dot{u}^2 + 2 \dot{u}\ w\ x)\ dy$$

$$+ \rho\ t\ h \int (v^2 w^2 + \dot{v}^2 + 2\ \dot{v}\ w\ y\ ) dx$$

$$+ m_2 (\ \dot{u}^2 + 2\ \dot{u}\ w\ (R+L) + u^2 w^2) + I2c(2w\dot{u}' + \dot{u}'^2)$$

$$+ m_2 (\dot{v}^2 + 2\ \dot{v}\ w(R+L) + v^2 w^2) + I2c(2w\dot{v}' + \dot{v}'^2)$$

$$(12)$$

Similar to the work of Turner and Chun in [11] a discretized model of the system is formed by assuming the elastic deformations of each of the appendages (relative to a body fixed undeformed state) are represented by a linear combination of comparison functions

$$\phi_i(z) = 1 - \cos(i\pi z/L) + .5\ (-1)^{i+1}\ (i\pi z/L)^2 \qquad (13)$$

where $z = x - R$

This assumed mode or shape function must be an admissable function that is used to approximate the deformation of the continuous beam being modeled. In order for these functions to be considered admissable, they must satisfy the geometric boundary conditions. These conditions are :

$$\phi\ (z=0) = 0 \qquad \text{and} \qquad \phi'\ (z=0) = 0$$

$$\text{since} \quad u(0,t) = u'(0,t) = 0$$

$$\text{and} \quad v(0,t) = v'(0,t) = 0$$

The transverse body fixed deformation curve of appendages 1 and 2 can be approximated as

$$u(t,z) = \sum_{i=1}^{p} U_i(t)\ \phi_i(z) \qquad (14)$$

where p is the number of modes to be analyzed and

$0 < z < L$

Likewise for appendages 3 and 4 ,

$$v(t,z) = \sum_{i=1}^{p} V_i(t) \quad \phi(z) \tag{15}$$

The assumed modes method leads to a generalized parameter model, where the $U_i(t)$ and $V_i(t)$ are the generalized displacement coordinates. Taking the first derivative of equations (14) and (15) with respect to time yields :

$$\dot{u}(t,z) = \sum_{i=1}^{p} \dot{U}_i(t) \ \phi_i(z) \tag{16}$$

$$\dot{v}(t,z) = \sum_{i=1}^{p} \dot{V}_i(t) \quad \phi_i(z) \tag{17}$$

Squaring the above equations

$$\dot{u}^2(t,z) = \sum_{i=1}^{p} \sum_{j=1}^{p} \dot{U}_i \ \dot{U}_j \ \phi_i \ \phi_j \tag{18}$$

$$\dot{v}^2(t,z) = \sum_{i=1}^{p} \sum_{j=1}^{p} \dot{V}_i \ \dot{V}_j \ \phi_i \ \phi_j \tag{19}$$

The above expressions for the generalized coordinates and their time rates of change, equations (14),(15), (16), (17), (18), and (19) can be inserted into the expression for the

total kinetic energy of the system, equation (12).

Potential Energy

The potential energy of the system can be written as the sum of the gravitional and the strain energy or elastic potential energy.

$$V = V_G + V_{elastic} \tag{22}$$

Since the hub is assumed to be supported by an air bearing table (or in orbit) the gravitional potential will be neglected and the potential energy written in terms of the strain energy alone.

$$V = V_{elastic} \tag{17}$$

The total strain energy is the sum of the contributions from all four appendages. The expression for $V_{elastic}$ has been derived extensively in the literature as

$$V_{elastic} = 1/2 \; E \; I \int_0^L (u'')^2 \, dz$$

or for all four appendages (1=2, 3=4):

$$V_{elastic} = E\,I \int_0^L (\,u''\,)^2\,dz + E\,I \int_0^L (\,v''\,)^2\,dz \quad (22)$$

where u'' and v'' are the second partial derivatives of u and v with respect to z, respectively. E is the modulus of elasticity of the arms , also known as Young's modulus, and is a property of the material. I is the area moment of inertia, based only upon t and h as

$$I = 1/12\ h\ t^3$$

Again, the assumed modes method leads to the generalized parameter model with U(t) and V(t) as the generalized displacement coordinates and $\phi_i(z)$ as the admissable functions.

u'' and v'' become

$$u'' = \sum_{i=1}^{p} U_i(t)\ \phi_i''(z) \quad (23)$$

$$v'' = \sum_{i=1}^{p} V_i(t)\ \phi_i''(z) \quad (24)$$

where ϕ_i'' is the second partial derivative of ϕ with

respect to z, see equation (13).

$$\phi_i'' = (i\pi/L)^2 \cos(i\pi z/L) + (-1)^{i+1}(i\pi/L)^2 \qquad (25)$$

Utilizing equations (23) and (24) and after grouping terms and arranging in matrix notation , equation (22) becomes

$$V_{elastic} =$$

$$EI \left\{U\right\}^T \left[\int_0^L \phi_i'' \phi_j'' dz\right] \left\{U\right\}$$

$$+ EI \left\{V\right\}^T \left[\int_0^L \phi_i'' \phi_j'' dz\right] \left\{V\right\}$$

(26)

Recalling Lagrange's equation for the derivation of the differential equations of motion as

$$d\,(\partial L / \partial q_i)\,/\,dt - \partial L / \partial q_i = Q$$

i=1, . . . no. of generalized coordinates

where L = T - V and Q represents the nonconservative forces.

Applying this method to the expressions for the kinetic energy and potential energy of the free vibration system and neglecting the terms of order three or higher and writing the kinetic and potential energies in the form :

$$T = 1/2 \quad \dot{\bar{q}}^T \; M \; \dot{\bar{q}}$$

$$V = 1/2 \quad \bar{q}^T \; K \; \bar{q}$$

where M and K are called the mass and stiffness matrices, and $\bar{q} = [\,\theta,\ U_1\ .\ .\ .\ U_p,\ V_1\ .\ .\ .\ V_p\,]^T$

The elements of the mass matrix are given by :

$$M\,(1,1) = 2\,(I_c + I_{2c}\,)$$

$$M(1,i+1) = 2\,\rho t\,h \int_R^{R+L} x\,\phi\,dx + 2\,(R+L)\,\phi(x=R+L)\,m_2 + 2\,I_{2c}\,\phi'(x=R+L)$$

$$M(1,i+p+1) = 2\,\rho\,t\,h \int_R^{R+L} y\,\phi\,dy + 2\,m_2(R+L)\,\phi(y=R+L) + 2\,I_{2c}\,\phi'(y=R+L)$$

$$M(i+1,1) = M(1,i+1)$$

$$M(i+p+1,1) = M(1,i+p+1)$$

$$M(i+1,j+1) = 2\,\rho\,t\,h \int_R^{R+L} \phi_i\,\phi_j\,dx$$

$$+2\ m_2\ \phi_i(x=R+L)\ \phi_j(R+L)$$

$$+2\ I_{2c}\ (\phi_i'(x=R+L))\ (\phi_j'(x=R+L))$$

$$M(i+p+1,j+p+1) = 2\ \rho\ t\ h \int_R^{R+L} \phi_i\ \phi_j\ dy$$

$$+2\ m_2\ \phi_i(x=R+L)\ \phi_j(x=R+L)\ +\ 2\ I_{2c}\ \phi_i'(x=R+L)\ \phi_j'(x=R+L)$$

$$M(i+1,j+p+1) = M(i+p+1,j+1) = 0 \qquad (27)$$

where $i=1,\ldots p$ and $j=1,\ldots p$

The resulting mass matrix is symmetric and of order 2 p+ 1 where p is the number of modes being evaluated. The elements of the stiffness matrix follow from equation (26) as

$$K(1,1) = K(1,j+1) = K(1,j+1+p) = 0$$

$$K(i+1,1) = K(i+1+p,1) = 0$$

$$K(i+1,j+1) = 2\ E\ I \int_0^L \phi_i''(z)\ \ \phi_j''(z)\ \ dz$$

$$K(i+1+p,j+1+p) = 2\ E\ I \int_0^L \phi_i''(z)\ \ \phi_j''(z)\ \ dz$$

and

$$K(i+1, j+1+p) = K(i+1+p, j+1) = 0 \qquad (28)$$

The vector differential equation governing the motion of the uncontrolled system takes the familiar form

$$M \ddot{\bar{q}} + K \bar{q} = 0 \qquad (29)$$

where

$$\bar{q} = [\ \Theta\ ,\ U_1\ .\ .\ .\ U_p,\ V_1\ .\ .\ .\ V_p\ ]^T$$

note: by assuming anti-symmetric motion of the appendages the number of generalized coordinates is reduced by 2 p .

The next step in the analysis is to solve the differential equation (29) to obtain the response of the uncontrolled system. The method used was to solve the familiar eigenvalue problem,

$$K \bar{x} = \lambda M \bar{x} \qquad (30)$$

for the eigenvalues and eigenvectors for the system. The first 10 modes were analysed and the resulting eigenvalues and eigenvectors are listed in Appendix A along with a plot of the first seven of those modes depicting the relative motion of the free arms. The modes appear as pairs, very close in frequency, depicting what Junkins in [4] calls 'opposition' and 'unison' modes. The opposition modes are

simple cantilever beam modes characterized by the adjacent beams moving in opposition. He states the unison modes are perturbed cantilever modes, with just slightly higher frequencies, with all four beams moving in unison and the hub having non-zero rotation. The rotation of the hub is to counter the unison movement of the appendages and point masses to conserve angular momentum in the system. The numerical values of the frequencies for the first seven modes are very close to those listed by Junkins in [4].

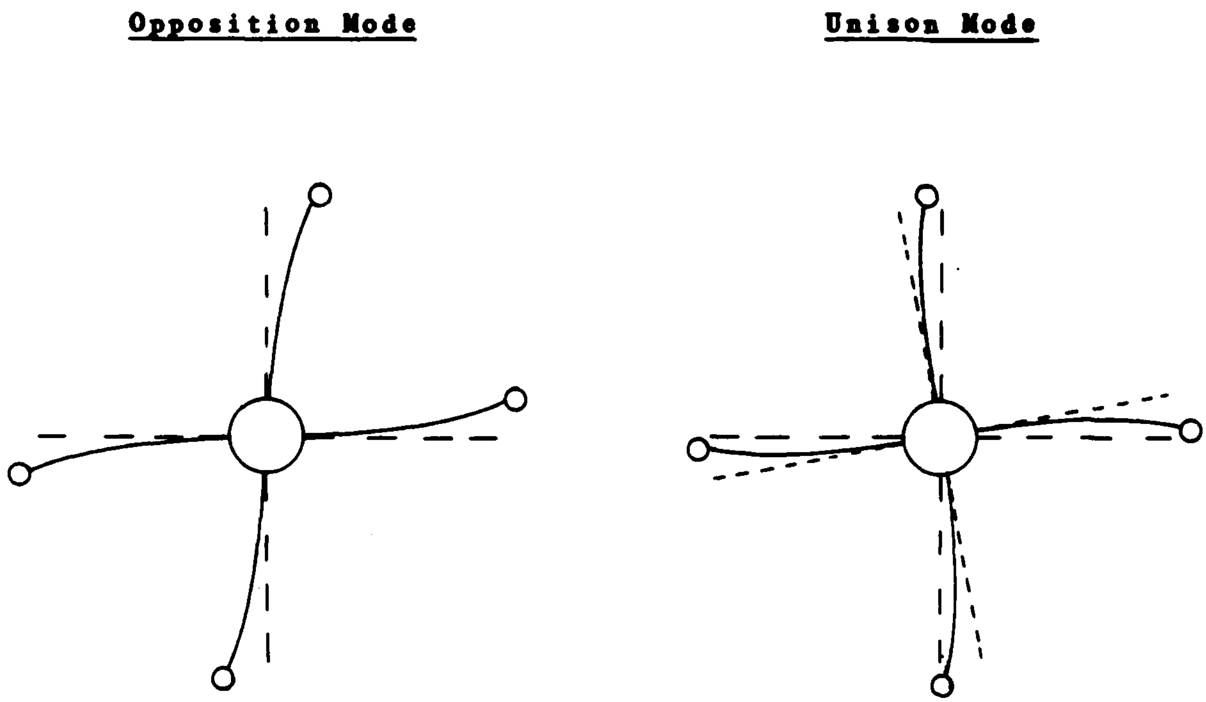

Figure 4. Opposition and Unison Modes

CONTROLLED SYSTEM

The equation of motion for the controlled linear system can be written in the general form of

$$M \ddot{\bar{q}} + C \dot{\bar{q}} + K \bar{q} = B \bar{\mu} \qquad (31)$$

where

M is the N x N (N=2p + 1) symmetric and positive definite mass matrix derived in the previous section.

C is the N x N structural damping matrix.

K is the N x N symmetric positive semi-definite stiffness matrix.

B is an N x m control matrix.

$\bar{q}$ is the vector, dimensionN, of generalized coordinates derived from the assumed modes method of the previous section.

$\bar{\mu}$ is the control vector of dimension m.

Only the case of linear output feedback control, with local position and velocity measurements available, will be considered here. In this case the gain matrices are constant such that

$$\bar{\mu} = -G_1 H_1 \bar{q} - G_2 H_2 \dot{\bar{q}} \qquad (32)$$

where H_1 and H_2 are the sensor matrices that representthe

linear relationship of the locally measured position and velocity. Substituting equation (32) into equation (31) yields the closed loop system equation

$$M \ddot{\bar{q}} + C^* \dot{\bar{q}} + K^* \bar{q} = 0 \tag{33}$$

where

$$C^* = C + B\ G_2\ H_2$$

or since for the problem examined zero system damping has been assumed:

$$C^* = B\ G_2\ H_2$$

and

$$K^* = K + B\ G_1\ H_1$$

The next step is to solve the differential equation (33) governing the controlled system to obtain the dynamical response. Equation (33) is a second order matrix differential equation representing the closed loop system. This second order differential equation may be written in first order form as

$$A \dot{\bar{\psi}} = D \bar{\psi} \tag{34}$$

where

$$A = \begin{bmatrix} -M & 0 \\ 0 & -M \end{bmatrix} \qquad D = \begin{bmatrix} 0 & -M \\ K^* & C^* \end{bmatrix}$$

and

$$\bar{\psi} = \begin{Bmatrix} \bar{q} \\ \dot{\bar{q}} \end{Bmatrix}$$

This first order differential equation can be solved by assuming a solution of the form

$$\bar{\psi} = \bar{\sigma}\, e^{\lambda t} \tag{35}$$

and solving for the eigenvalues and right and left eigenvectors of the controlled system. Since matrix A and D are of order 2 N , there will also be 2 N eigenvalues and the right and left eigenvectors will be of order 2N. They are found from

$$\text{right} : \lambda_i\, A\, \bar{\sigma}_i = D\, \bar{\sigma}_i$$

$$\text{left} : \lambda_i\, A^T\, \bar{\gamma}_i = D^T\, \bar{\gamma}_i$$

$$\text{where} \quad i=1, \ldots 2N \tag{36}$$

The solution to the uncontrolled system was found using the values for G_1 and G_2 found optimum by Junkins in [4] and using the same expressions for B, H_1, and H_2.

$$G_1 = \begin{bmatrix} 8.60 & -1.08 & -3.09 & -1.08 & -3.09 \\ 21.80 & -1.74 & -4.94 & -1.74 & -4.94 \\ 21.90 & -1.74 & -4.94 & -1.74 & -4.94 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} -.238 & -.016 & -.050 & -.016 & -.050 \\ .643 & .233 & .119 & -.015 & .028 \\ .643 & -.015 & .028 & .233 & .119 \end{bmatrix}$$

Following the same configuration of the Draper model as Junkins, torque actuators were admitted on the central hub and at some position (z) on each appendage. The actuators consisted of a torque μ_1 applied to the hub, a torque μ_2 at z=L/2 for arms 1 and 2 and a torque μ_3 applied at z=L/2 for the third and fourth appendages. Matrix B is then

$$B = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2\ \phi'(z=L/2) & 0 \\ 0 & 0 & 2\ \phi'(z=L/2) \end{bmatrix} \begin{Bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{Bmatrix} \tag{37}$$

where $\phi'(z)$ as stated before is $d(\phi(z))/dz$ and the number of elements of ϕ' is equal to the number of modes being examined. B is an N x 3 matrix.

Co-located position and velocity sensors are located at the same positions for appendages 1 and 2 and appendages 3 and 4. This has the effect of rendering $H_1 = H_2 = H$.

$$H = \begin{bmatrix} 1 & \bar{0}^T & \bar{0}^T \\ 0 & \phi^T(z=L/2) & \bar{0}^T \\ 0 & \phi^T(z=L) & \bar{0}^T \\ 0 & \bar{0}^T & \phi^T(z=L/2) \\ 0 & \bar{0}^T & \phi^T(z=L) \end{bmatrix} \tag{37}$$

hence H is a 5 x N matrix.

Inserting the expressions for G_1 , G_2 , B and H into

equation (33) and then (34) allows the solution of equations (36) for the eigenvalues and the corresponding right and left eigenvectors. The actual numerical data used for the parameters of the system are listed in Table 1 and were taken from [4]. The only changes made were to standardize all the units of length to feet to avoid confusion and numerical error. The results for the first 10 modes are listed in Appendix B.

Hub radius, R	1 ft
Rotary Inertia of Hub	8 slug-ft^2
Mass density of beams, ρ	5.22 slug-ft^3
Modulus of Elasticity, E	1.584E+09 lb/ft^2
Appendage thickness, t	.0104166 ft
Appendage height, h	.5 ft
Appendage length, L	4.0 ft
Tip mass	.156941 slug
Rotary Inertia of tip mass	.0018 slug-ft^2

Table 1. Configuration Parameters for Draper Model

III. OPTIMIZATION TECHNIQUE

The parameter optimization technique or method used to minimize the mass of the 'Draper /RPL Configuration' model involves the eigenspace optimization approach developed by Junkins in [4] and is based upon the work by Plaut and Huseyin in [7]. As in any optimization scheme , an objective function is sought to be minimized subject to certain imposed equality and inequality constraints. 'An algorithm that can effectively optimize a global structural and control parameter vector (over some admissible set, to minimize some optimality criteria and satisfy constraints stated as functions of that vector and the eigenvalues) provides a direct method for controller/structure optimization problems.'[4]

SOLUTION PROCEDURE

Let $\bar{d}$ be a vector of global structural and control parameters such that

$$M = M(\bar{d}) \quad , \quad C^{*} = C^{*}(\bar{d}) \quad , \quad K^{*} = K^{*}(\bar{d}) \qquad (39)$$

it follows that the eigenvalues and eigenvectors will also be a function of $\bar{d}$.

$$\lambda_i\,(\bar{d}) \quad , \quad \bar{\sigma}_i\,(\bar{d}) \quad , \quad \bar{\gamma}_i\,(\bar{d}) \qquad (40)$$

The eigenvalues of a system can be a highly nonlinear function of the structural and control parameters and may contain bifurcation points at repeated roots. Therefore,the constraints should include a measure of eigenvalue sensitivity to prevent encountering these singular points. Another problem described by Junkins in [4] that can be encountered is in attempting to place eigenvalues at exact postions or 'pole placement'. Therefore, instead of attempting to prescribe an exact location for an eigenvalue, a region (in the left half plane) will be specified.

The technique used in the optimization is based upon the necessity to effectively compute the eigenvalue sensitivities to changes in the structural parameters, or better stated as the partial derivatives of the eigenvalues denoted below by:

$$\partial\lambda_i \,/\, \partial d_l \qquad \begin{array}{l} i=1 \ . \ . \ . \ N \\ l=1 \ . \ . \ . \ \text{no. of parameters} \end{array} \tag{41}$$

for the generalized eigenvalue problem of equation (36).

Plaut and Huseyin in [7] and Rogers in [8] present an excellent derivation of equation (41), the results of which are presented below

$$\partial\lambda_i/\partial d_l = \bar{\gamma}_i^T \, [\partial D/\partial d_l - \lambda_i \partial A/\partial d_l \,] \, \bar{\sigma}_i \tag{42}$$

The sensitivities of the eigenvalues can be computed once the

eigenvalues and eigenvectors have been found. Although the calculations do not appear to be extremely difficult, they are tedious and tedious work is best left to a machine, in this case the Cyber computer.

Expanding equation (42) yields :

$$\partial\lambda_i/\partial d_1 = \bar{\gamma}_i^T\, \partial D/\partial d_1\, \bar{\sigma}_i - \lambda_i \bar{\gamma}_i^T \partial A/\partial d_1 \bar{\sigma}_i \tag{43}$$

Remembering that

$$D = \begin{bmatrix} 0 & M \\ K^* & C^* \end{bmatrix} \qquad\qquad A = \begin{bmatrix} -M & 0 \\ 0 & -M \end{bmatrix}$$

Then

$$\partial D/\partial d_1 = \begin{bmatrix} 0 & \partial M/\partial d_1 \\ \partial K^*/\partial d_1 & \partial C^*/\partial d_1 \end{bmatrix}$$

$$\partial A/\partial d_1 = \begin{bmatrix} -\partial M/\partial d_1 & 0 \\ 0 & -\partial M/\partial d_1 \end{bmatrix} \tag{44}$$

expanding the above terms and using the chain rule extensively, the partial derivatives needed in (44) can be calculated.

$$K^* = K(\bar{d}) + B(\bar{d})\ G_1(\bar{d})\ H(\bar{d})$$

$$\partial K^*/\partial d_1 = \partial K/\partial d_1 + \partial B/\partial d_1\ G1\ H + B\ \partial G1/\partial d_1\ H$$

$$+ \ B\ G1\ \partial H/\partial d_1 \qquad (45)$$

and $C^* = B\ G_2\ H$

$$\partial C^*/\partial d_1 = \partial B/\partial d_1\ G2\ H + B\ \partial G2/\partial d_1\ H + B\ G2\ \partial H/\partial d_1$$

$$(46)$$

Once the initial structural and control parameters are specified, the mass and stiffness matrices can be computed and fed to an algorithm for computing the eigenvalues and eigenvectors (left and right). With a few additional moderate calculations, the sensitivities of the eigenvalues can also be computed. Therefore, the requirement to effectively calculate the eigenvalue sensitivities has been fulfilled and the task of accomplishing the stated objective can be tackled.

The objective is to minimize the mass of the structure given an initial design with an optimum control system already developed for the initial design, by optimizing the structural parameters. The objective function is subject to the constraints of maintaining specified natural frequencies and damping factors constant (or within specified tolerances). The mass of the hub will be held constant as will the tip mass at the end of each appendage, such that

only the mass of the appendages will be minimized. Restated, the objective is to minimize

$$M = \sum_{i=1}^{4} \rho \ t_i \ h_i \ L_i \tag{47}$$

subject to specified natural frequencies and damping factors. The natural frequencies and damping factors are related to the structural parameters through the eigenvalues as

$$\omega_{ni} = [\ (\text{Real } \lambda_i\ )^2 + (\text{Imag } \lambda_i)^2\ ]^{1/2}$$

$$\xi_i = -\text{Real } \lambda_i \ / \ \omega_{ni}$$

where $\lambda_i = \lambda_i(\bar{d})$

(48)

The role of the eigenvalue sensitivities becomes readily apparent when the gradients or partial derivatives of the natural frequencies and damping factors with respect to the structural parameters are calculated using the chain rule.

$$\partial\omega_i/\partial d_1 = [\partial\text{Real } \lambda_i/\partial d_1 + \partial\text{Imag}\lambda_i/\partial d_1]/ \ \omega_{ni}$$

$$\partial\xi/\partial d_1 = -[\partial\text{Real } \lambda_i/\partial d_1] \ / \ \omega_{ni}$$

$$+ \ [\ \partial\text{Real } \lambda_i/\partial d_1 + \partial\text{Imag } \lambda_i/\partial d_1\ ] \ / \ \omega_{ni}{}^{3/2}$$

(49)

If the structural parameters are considered to be the

thickness, t, the height, h, and the length, L, of each appendage then the parameter vector $\bar{d}$ becomes

$$d = [t_1 \ldots t_4,\ h_1 \ldots h_4,\ L_1 \ldots L_4]^T \tag{50}$$

In the problem considered here, all four appendages are kept structurally identical, such that the parameter vector is reduced to only three elements

$$\bar{d} = [\ t \quad h \quad L\ ]^T \tag{51}$$

If a need were to arise to change certain appendages while holding others constant or to change all four, but weighted differently, then the dimension of $\bar{d}$ would increase accordingly. Referring back to equations (45) and (46) the gain matrices G_1 and G_2, are functions of the control gains alone and therefore the partial derivatives of G_1 and G_2 with respect to the current parameter vector are zero. That is

$$\partial G_1/\partial \bar{d} = 0$$

and

$$\partial G_2/\partial \bar{d} = 0 \tag{52}$$

Also the control matrices B and H are not functions of t or h, therefore

$$\partial B/\partial t = \partial B/\partial h = 0$$

$$\partial H/\partial t = \partial H/\partial h = 0 \qquad (53)$$

This reduces the number of calculations required when computing the eigenvalue sensitivities and hence the gradients of the natural frequencies and damping factors.

Therefore, the partial derivatives of matrices A and D reduce to calculating the partial derivatives of the mass and stiffness matrices for elements t and h of the parameter vector $\bar{d}$.

For element 1, the thickness t :

$\partial M(1,1)/\partial t = 4/3\ \rho\ h\ I_1$ (I_1 is defined in ch.2)

$\partial M(1,j+1)/\partial t = 2\ \rho\ h\ Px_j$

$\partial M(1+j,1)/\partial t =$ ''

$\partial M(1+j+p,1)/\partial t =$ ''

where j=1,p and p is the number of modes.

and $$Px_j = \int_R^{R+L} x\ \phi_j(x)\ dx \qquad (54)$$

$\partial M(i+1,j+1)/\partial t = 2\ \rho\ h\ P_{i,j}$

$\partial M(i+1+p,j+1+p)/\partial t =$ ''

where i=1,p and j=1,p

and $$P_{i,j} = \int_0^L \phi_i(z)\ \phi_j(z)\ dz \tag{55}$$

other elements of the gradient of the mass are zero.

Similarly for the stiffness

$\partial K(i+1,j+1)/\partial t = .5\ E\ h\ t^2\ P_{i,j}''$

$\partial K(i+1+p,j+1+p)/\partial t =$ ''

where $$P_{i,j}'' = \int_0^L \phi_i''(z)\ \phi_j''(z)\ dz$$

(56)

and the other elements are zero.

For element 2, the height, h :

$\partial M(1,1)/\partial h = 4/3\ \rho\ t\ I_1$

$\partial M(1,j+1)/\partial h = 2\ \rho\ t\ \ Px_j$

$\partial M(1,j+1+p)/\partial h =$ ''

$\partial M(1+j,1)/\partial h =$ ''

$\partial M(j+1+p,1)/\partial h =$ ''

$j=1,p$ (57)

$\partial M(i+1,j+1)/\partial h = 2\ \rho\ t\ P_{i,j}$

$\partial M(i+1+p,j+1+p)/\partial h =$ '' (58)

all other elements of M are zero.

For the stiffness :

$$\partial K(i+1,j+1)/\partial h = 1/6\ E\ t^3\ P_{i,j}''$$

$$\partial K(i+1+p,j+1+p)/\partial h = \quad ''$$

$$i=1,p \quad , \quad j=1,p \qquad (59)$$

and all other elements of K are zero.

For the third element, L , the calculations become considerably more involved as B and H and ϕ are functions of z which ranges from 0 to L.

$$\partial D/\partial L = \begin{bmatrix} 0 & \partial M/\partial L \\ \partial K^*/\partial L & \partial C^*/\partial L \end{bmatrix}$$

$$\partial A/\partial L = \begin{bmatrix} -\partial M/\partial L & 0 \\ 0 & -\partial M/\partial L \end{bmatrix} \qquad (60)$$

Expanding,

$$\partial M(1,1)/\partial L = 4\rho\ t\ h\ (R+L)^2 + 8\ m_2 (R+L)$$

$$\partial M(1,j+1)/\partial L = 2\ \rho\ t\ h\ \partial Px_j/\partial L$$

$$+ 2\ m_2\ \phi_j(L) + 2\ I_{2c}\ \partial\ \phi_j'(L)/\partial L$$

$$\partial M(1,j+1+p)/\partial L = \quad ''$$

$$\partial M(j+1,1)/\partial L = \quad ''$$

$$\partial M(j+p+1,1)/\partial L = \quad ''$$

$$j=1,p$$

and $\partial \phi_j'(L)/\partial L = -(-1)^{j+1}\ (j\pi/L)^2$

$$\partial M(i+1,j+1)/\partial L = 2\rho\ t\ h\ \partial P_{i,j}/\partial L$$

$$+ 2\ I_{2c}\ \partial(\phi_i'(L)\ \phi_j'(L))/\partial L$$

$$\partial M(i+1+p, j+1+p)/\partial L = \quad ''$$

where $\partial(\phi_i' \ \phi_j')/\partial L = -2 \ (i\pi \ j\pi)^2 \ (-1)^{i+1} \ (-1)^{j+1} \ /L^3$

(61)

Expanding the remaining terms in matrix D yields :

$$\partial K^*/\partial L = \partial K/\partial L + \partial B/\partial L \ G_1 \ H + B \ G_1 \ \partial H/\partial L$$

$$\partial C^*/\partial L = \partial B/\partial L \ G_2 \ H + B \ G_2 \ \partial H/\partial L$$

$$\partial K(i+1, j+1)/\partial L = 1/6 \ E \ h \ t^3 \ \partial P_{i,j}''/\partial L$$

$$\partial K(i+1+p, j+1+p)/\partial L = \quad ''$$

where

$$P_{i,j}'' = \int_0^L \phi_i''(z) \ \phi_j''(z) \ dz$$

(62)

$$\partial B(i+1, 2)/\partial L = 2 \ \partial\phi_i'(z=L/2)/\partial L$$

$$\partial B(1+1+p, 3)/\partial L = \quad ''$$

where

$$\partial\phi_i'(z=L/2)/\partial L = -(i\pi/L^2) \ \sin(i\pi/2)$$

(63)

$$\partial H(1, j+1)/\partial L = \partial\phi_j(z=L/2)/\partial L = 0$$

$$\partial H(4, j+1+p)/\partial L = \quad ''$$

$$\partial H(3, j+1)/\partial L = \partial\phi_j(z=L)/\partial L = 0$$

$$\partial H(5, j+1+p)/\partial L = \quad ''$$

The above equations, although lengthy when expanded, involve repitition and matrix multiplication which are ideally suited for a computer.

DESCRIPTION OF COMPUTER PROGRAMS

Utilizing the above development, a computer program was written to calculate the gradients of the natural frequency and the damping factor for each eigenvalue. The program is listed in Appendix C . It begins by calculating the mass and stiffness matrices for a given $\bar{d}$, initial values of t, h, and L. The control matrices, B and H, are computed and combined with the given gain matrices to form the A and D matrices of

$$D\ \bar{\sigma} = \lambda\ A\ \bar{\sigma}$$

and

$$D^T\ \bar{\gamma} = \lambda\ A^T\ \bar{\gamma} \tag{50}$$

to compute the eigenvalues and left and right eigenvectors. Once A and D are formed they are input to an IMSL subroutine called 'EIGZF' along with the appropriate dimensions. EIGZF returns the eigenvalues and corresponding eigenvectors. The transpose of A and D are computed and also input to EIGZF, which returns the left eigenvectors.

Subroutine CALCBA is called to compute the gradients of the A and D matrices with respect to the input structural parameters. This information along with the eigenvalues and eigenvectors is passed to another subroutine called GRADNT, which as the name implies, computes the gradients or sensitivities of the natural frequency and damping factors. The eigenvalue sensitivities are calculated by expanding

equation (42) and using the input values for the eigenvalues, left and right eigenvectors, and the gradients of matrices A and D. The partial derivatives of the natural frequencies and damping factors can now be caluculated using equation (49).

There are many standard software optimization algorithms available that allow the user to minimize a stated objective function subject to specified equality and inequality constraints. The algorithm used here was one called CONMIN (constrained minimization) which is described in [12]. It is a FORTRAN program in subroutine form for constrained function minimization. The user must provide a main program to call CONMIN and evaluate the objective and constraint functions along with the associated gradients. CONMIN then uses this information in conjunction with the method of Feasible Directions to solve the constrained problem.

A main program was written to call CONMIN and perform the necessary calculations. The program initializes all variables common to CONMIN and then prompts the user for certain inputs. Those inputs are DALFUN, DABFUN, CT, THETA which specify the accuracy, tolerances , and how far off to move from the initial values of the decision variables (in this case the decision variables are the structural parameters). Next the user is asked to input the initial values for the decision variables. Subroutine CALOBJ is

called which calculates the objective function

$$OBJ = 4 \rho t h L \tag{64}$$

where ρ is the mass per unit volume

Subroutine CALOBJ calls the program described earlier to provide the eigenvalues and gradients of the natural frequencies and damping factors (which are necessary since the equality constraints are the natural frequency and damping factor and the objective is to minimize the mass while holding them constant). The constraints then become

$$G(1) = \omega_1 - \omega_o$$
$$G(2) = \xi_1 - \xi_o \tag{65}$$

where ω_1 and ξ_1 are the original values of the natural frequency and damping factor calculated for the initial structural parameters and the gains found optimum by Junkins in [4]. These are the values desired to be held constant while ξ_o and ω_o are the values re-computed during each iteration of CONMIN and evaluated. The gradients of the natural frequency and damping factor are also passed to CONMIN to provide convergence information. The constraint gradients provide a direction for the process such that the value of the objective function continues to decrease while satisfying the constraints G(1) and G(2). The ideal performance of obtaining G(1) or G(2) equal to zero can seldom be obtained in a digital computer, therefore CONMIN

uses a constraint thickness parameter to define the region over which G(1) or G(2) can be assumed zero for optimization purposes. This is one of the prompted input values called CT, so the program allows the user to input how stringent the process is to be. The constraints, G, are considered active if they are

CT. LE . G(j). LE . ABS (CT)

where CT is the constraint thickness parameter. If they are no longer active, the specified upper and lower bounds on the decision variables (structural parameters) may become active. When the objective function can no longer decrease, the process is terminated.

Therefore, the result should be a structure that begins with a specified feasible set of structural parameters, optimizes the control system , and then optimizes the structural design. This procedure can be repeated until the desired mission objectives are achieved.

IV. ANALYSIS OF RESULTS

The objective of this report is to minimize the mass of the appendages while constraining either the fundamental frequency or another natural frequency to predetermined values. The gradients of the eigenvalues were calculated, with respect to each element of $\bar{d}$, for the firstten eigenvalues. This information was used while examining four different cases. Those cases are: constrainingthe fundamental frequency to the initial value found using Junkins' optimum control law, constrain the third natural frequency to an initial value, constrain the third natural frequency to an acceptable region, and constraining the third natural frequency to a specified region while also constraining the fundamental frequency.

GRADIENTS AND EXPECTATIONS

The program used to call CONMIN was modified to print the gradients of the eigenvalues and was run separately. The gradient of the first eigenvalue for all three parameters was found to be approximately -2.0 E-10. This would imply that the 'surface' is very flat and convergence to a new minima difficult. Therefore, one would expect very little change in the objective function for an input of initial values and difficulty in convergence if a non-feasible solution was input.

The second eigenvalue is the conjugate of the first, so

the gradient of the third eigenvalue was examined. The gradient with respect to the thickness was found to be very large, approximately 2200, in comparison to the other elements, 14.52 for h and -6.45 for L. Therefore,one would expect the third eigenvalue to be very sensitive to a change in the thickness and less sensitive to a corresponding change in the height or length. Since the objective function is linear with respect to each of the three parameters, a large change in the height or length would reduce its value while having less effect upon the eigenvalue than a corresponding change in the thickness. Convergence for the third eigenvalue can be expected to a much lower value for h or L with t remaining approximately the same. The value of the objective function should show a corresponding decrease.

CASE 1

The first case examined constrainedthe fundamental frequency and damping coefficient to the values found using the nominal thickness, height, and length given by Junkins in [4] and using the gains he found to be optimum. The first attempt input those nominal values for t, h, and L and as expected, very little change occurred as the program converged. The resulting values were :

$M = .434780$ $\Delta M = .000217$

$t = .010411$ $\Delta t = .0000056$

$h = .50$ $\Delta h = 0$

$L = 4.0$ $\Delta L = 0$

When other values for t, h, or L were input, the process would not converge and a feasible solution could not be found. Since the input values for t, h, and L did not return values for ω_1 and ξ_1 that satisfied the constraints and the gradients were nearly zero, convergence could not occur and after 10 iterations a feasible solution could not be found.

CASE 2

The natural frequency and damping coefficient of the third eigenvalue were constrained to be equal to the values found using the nominal t, h, and L and the gains found by Junkins. Although the gradients of λ_3 are much greater than those of λ_1 , the program still would not reduce the objective function any significant amount.

	Input	Final
t =	.0104166	.01044985
h =	0.50	.49152336
L =	4.0	3.9947516
M =	.434997	.428425

ΔM = .00657 , 1.5%

When other values for t, h, and L were tried the program would not satisfy the constraints and a feasible solution could not be found. Different values were tried for the constraint thickness parameter and larger push off factors (theta) and still the program would not converge. As an example, the following values were attempted :

$t = .0097$ $h = .5$ $L = 4.0$

$t = .02$ $h = .5$ $L = 4.0$

$t = .015$ $h = .4$ $L = 3.7$

Appendix D provides a more detailed listing of some of this output. These initial values do not satisfy the constraints and represent an initial non-feasible solution, which CONMIN states it can handle. Values for CT = -.1, -.2, -1.0, -2.0 were tried with the above values for t, h, and L. The constraint thickness specifies a region for which the constraints may be considered satisfied, so by increasing the value of CT, the program is allowed greater flexibility to solve the problem. However, even with the abnormally large values of CT (-1.0, -2.0) the program would not converge to a feasible solution. The problem was due to CONMIN's inability to solve equality constraints (although the user's manual states that it can) and with only three parameters allowed to change, the thickness , height, and length, there was not enough freedom for the program to search for the optimum solution and satisfy the constraints.

CASE 3

To alleviate the problems experienced with the equality constraint, a region was specified in which to constrain the third natural frequency. The initial value of ω_3 was 4.38 and the region specified by the new constraints was ± .2 from the original value. The constraint on the damping

coefficient was removed to allow greater freedom of movement for the program. Therefore, the equality constraint imposed in Case 2 was replaced by an inequality constraint. Several different values for t, h, and L were input along with different values for the constraint thickness, CT, and DELFUN, the minimum relative change in the objective function to indicate convergence. Different values for the push off factor, Theta, were also combined with the above. Table 2 lists the values input and the final values for some of the cases input. All converged to local minimums and satisfied the constraints for each input. A more detailed listing of these results are presented in Appendix E . Many solutions were found which satisfied the constraints and reduced the mass considerably. This was due to the steepness of the curve representing $\partial\lambda_3/\partial t$ and also the second partial derivative of M was equal to zero for all three parameters.

Instability in the fundamental frequency was encountered when the constraint thickness parameter was increased to -2.0. This increase provided additional flexibility in the optimization process and reduced the mass even more as shown in Table 2. However, this caused λ_1 to move to an unstable region because there was no constraint imposed to keep it in the left half plane. Since more than one relative minima exists and some of these represent unstable solutions, a constraint was imposed on the real part of λ_1.

INPUT				RESULT				
t	h	L	CT	t	h	L	M	λ_1
.0104	.5	3.7	-2.0	.012	.1878	3.58	.1712	+1.815
.012	.5	4.0	-2.0	.0153	.1434	3.87	.1775	+.4146
.0104	.5	4.0	-0.1	.012	.3189	3.937	.31835	-.095
.012	.5	4.0	-1.0	.01275	.2695	3.928	.28183	-.099

Table 2. Parameters for Case 3.

CT is the constraint thickness parameter and λ_1 represents the real part of the first eigenvalue.

CASE 4

Unstable vibration at the fundamental frequency is not acceptable, hence another constraint was imposed to maintain the real part of λ_1 in the left half plane. ω_3 was still constrained to remain in the previously specified region.

Several attempts were made to minimize the mass with these new constraints. Some converged, but did not reduce the mass more than about 3%, while others would not converge at all. When values other than the nominal values for t, h, and L were input, .0104166, .5, 4.0 respectively, the program would not converge. Several attempts were tried with different values for CT, DELFUN, DABFUN, and THETA, but the program still would not converge. The constraint that was not satisfied most often was the constraint on ω_3. The program would reduce the mass and maintain λ_1 in the left half plane, but ω_3 tended towards a value of 5.0-5.2. Therefore, the constraint on ω_3 was changed to maintain it in the region 4.2 - 5.2 and the program began to function much better.

The values found optimum in Case 3 were input with the new constraints and the results are shown in Table 3. The only convergence problem encountered is shown. Several combinations of values for CT, DELFUN, DABFUN, and THETA were tried, but to no avail. It was found that if the values were not close to a feasible solution with gradients of sufficient magnitude to provide direction, the program would satisfy one

of the constraints (maintain ω_3 in the specified region), but not the other (maintain λ_1 in the left half plane). The values input for t, h, and L represented a value for the real part of λ_1 of +1.815 which evidently was too far off to permit convergence, due to the small gradients of λ_1.

The optimum values were found to be as shown in Table 3 with the greatest reduction in appendage mass of 60.7%. A more detailed listing of this output is in Appendix F along with the eigenvalues and eigenvectors for the reconfigured structure.

INPUT				RESULT				
t	h	L	CT	t	h	L	M	λ_1
.0104	.5	4.0	-.1	.0127	.494	3.99	.4238	-.009
.0104	.5	4.0	-1.0	.0103	.483	3.99	.4167	-.010
.012	.1878	3.58	-.1	Would not satisfy				+1.8
.012	.1878	3.58	-1.0	the constraint on				+1.8
.012	.1878	3.58	-2.0	λ_1				+1.8
.0127	.2695	3.93	-.1	.0125	.2695	3.93	.2772	-.010
.0127	.2695	3.93	-1.0	.0125	.2695	3.93	.2772	-.0095
.012	.3189	3.94	-.1	.0118	.319	3.94	.3108	-.0099
.0104	.3	3.8	-1.0	.0119	.2997	3.79	.2841	-.0074
.0153	.1434	3.87	-.1	.0156	.1365	3.83	.17015	-.0089

Table 3. Results for Case 4.

V. RECOMMENDATIONS

Further study should be done to expand the approach used here to allow for varying more parameters. The appendages in this study were kept structurally identical which reduced the number of degrees of freedom in the optimization to three before the constraints were included. By changing the appendages to pairs of structurally identical beams, the parameter vector can be increased by a factor of two. This would permit greater freedom in the program for optimization of the parameters and minimization of the total mass while constraining the frequencies. The cross-sectional areas of the arms were kept constant from the root to the tip in this report, further study should include varying the cross-sectional area such that tapered beams are also included. The program developed here to call CONMIN should be expanded to include an algorithm to optimize the closed loop control law for the optimized structure. A continuation procedure could be used as an iterative technique in a similar manner to that used by Venkayya in [9] or by Junkins in [4]. The program should begin with a viable solution and iterate until the mission objectives can be accomplished with the resulting structure and control law. Another interesting expansion would be to solve the control and structure problem simultaneously.

VI. CONCLUSION

As stated by Junkins in [4] exact placement of eigenvalues is difficult and convergence unlikely, rather, specifying a particular region for the eigenvalue leads to a much better algorithm and convergence. By reducing the number of constraints and increasing the degrees of freedom in the problem, the program converged to a solution which minimized the mass of the structure while keeping the first natural frequency in a stable region and maintaining the third natural frequency in a specified region. By changing the constraint from an equality constraint to an inequality constraint, the algorithm functioned much better and more efficiently. The program developed in this report can be expanded to incorporate a control optimization algorithm by increasing $\bar{d}$ to include the elements of the gain matrices and the actuator and sensor locations. Incorporating control into the structural optimization problem can be accomplished effectively by increasing the number of degrees of freedom in the problem and using an iterative technique. This can be expanded, provided one has the computer resources, to a much higher dimensioned system. The eigenspace optimization approach developed by Junkins in [4] and expanded here with a constrained function minimization program, provides a viable approach to the problem of incorporating optimal control into the structural design process.

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APPENDIX A

EIGENVALUES AND EIGENVECTORS OF FREE SYSTEM

The eigenvalues and eigenvectors of the free (uncontrolled) system were found by using program PHICAL, which is listed in subroutine form in Appendix C, to calculate the mass and stiffness matrices. A modified version of program MEIGLR, listed in subroutine form in Appendix C, uses the mass and stiffness matrices to calculate the eigenvalues and corresponding eigenvectors. The first 10 eigenvalues and eigenvectors are listed. The first one represents the rigid body mode, while the remaining nine represent the modes published by Junkins in [4] and resemble those of a cantilever beam with a lumped mass at the end. A plot of these modes is also included. The actual deflection in the v direction is of opposite sign to computer output.

EIGENVALUE 1 IS:

.00000000E+00
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.00000000E+00
.00000000E+00

AND ITS EIGENVECTOR IS:

.10000000E+01	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00
.00000000E+00	.00000000E+00

EIGENVALUE 2 IS:

.19123573E+02
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.43730507E+01
.00000000E+00

AND ITS EIGENVECTOR IS:

-.38933111E-12	.00000000E+00
-.10000000E+01	.00000000E+00
-.29368652E-02	.00000000E+00
-.12880784E-01	.00000000E+00
.36673167E-03	.00000000E+00
-.19815906E-02	.00000000E+00
.10000000E+01	.00000000E+00
.29368652E-02	.00000000E+00
.12880784E-01	.00000000E+00
-.36673167E-03	.00000000E+00
.19815906E-02	.00000000E+00

EIGENVALUE 3 IS:

.62560428E+02
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.79095150E+01
.00000000E+00

AND ITS EIGENVECTOR IS:

.10000000E+01	.00000000E+00
-.93098297E+00	.00000000E+00
.71130104E-02	.00000000E+00
-.11744341E-01	.00000000E+00
.98375566E-03	.00000000E+00
-.18661096E-02	.00000000E+00
-.93098297E+00	.00000000E+00
.71130104E-02	.00000000E+00
-.11744341E-01	.00000000E+00
.98375566E-03	.00000000E+00
-.18661096E-02	.00000000E+00

EIGENVALUE 4 IS:

.26752296E+04
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.51722621E+02
.00000000E+00

AND ITS EIGENVECTOR IS:

-.13419288E-11	.00000000E+00
-.10000000E+01	.00000000E+00
-.28147448E+00	.00000000E+00
.11479128E-02	.00000000E+00
-.16285661E-01	.00000000E+00
.15240707E-02	.00000000E+00
.10000000E+01	.00000000E+00
.28147448E+00	.00000000E+00
-.11479128E-02	.00000000E+00
.16285661E-01	.00000000E+00
-.15240707E-02	.00000000E+00

EIGENVALUE 5 IS:

.28177824E+04
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.53082788E+02
.00000000E+00

AND ITS EIGENVECTOR IS:

-.35977156E-01	.00000000E+00
.10000000E+01	.00000000E+00
.27192910E+00	.00000000E+00
-.20087475E-02	.00000000E+00
.15639824E-01	.00000000E+00
-.15868837E-02	.00000000E+00
.10000000E+01	.00000000E+00
.27192910E+00	.00000000E+00
-.20087475E-02	.00000000E+00
.15639824E-01	.00000000E+00
-.15868837E-02	.00000000E+00

EIGENVALUE 6 IS:

.25682245E+05
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.16025681E+03
.00000000E+00

AND ITS EIGENVECTOR IS:

.267774115E-10	.00000000E+00
.10000000E+01	.00000000E+00
-.30251849E+00	.00000000E+00
-.20012248E+00	.00000000E+00
.997650065E-02	.00000000E+00
-.22870335E-01	.00000000E+00
-.99999999E+00	.00000000E+00
.30251849E+00	.00000000E+00
.20012248E+00	.00000000E+00
-.99765064E-02	.00000000E+00
.22870335E-01	.00000000E+00

EIGENVALUE 7 IS:

.25949087E+05
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.16108720E+03
.00000000E+00

AND ITS EIGENVECTOR IS:

.85234335E-02	.00000000E+00
.10000000E+01	.00000000E+00
-.31237361E+00	.00000000E+00

-.20340675E+00	.00000000E+00
.10519143E-01	.00000000E+00
-.23206053E-01	.00000000E+00
.99999999E+00	.00000000E+00
-.31237361E+00	.00000000E+00
-.20340675E+00	.00000000E+00
.10519143E-01	.00000000E+00
-.23206053E-01	.00000000E+00

EIGENVALUE 8 IS:

.11410779E+06
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.33779845E+03
.00000000E+00

AND ITS EIGENVECTOR IS:

-.16882015E-09	.00000000E+00
-.99999996E+00	.00000000E+00
.73378994E+00	.00000000E+00
-.41778865E+00	.00000000E+00
-.43354098E+00	.00000000E+00
.52687559E-01	.00000000E+00
.10000000E+01	.00000000E+00
-.73378997E+00	.00000000E+00
.41778866E+00	.00000000E+00
.43354100E+00	.00000000E+00
-.52687561E-01	.00000000E+00

EIGENVALUE 9 IS:

.11453340E+06
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.33842783E+03
.00000000E+00

AND ITS EIGENVECTOR IS:

-.82007604E-02	.00000000E+00
.99999996E+00	.00000000E+00
-.72262577E+00	.00000000E+00
.41881917E+00	.00000000E+00
.43060762E+00	.00000000E+00
-.52829253E-01	.00000000E+00
.10000000E+01	.00000000E+00
-.72262580E+00	.00000000E+00

.41881918E+00	.00000000E+00
.43060764E+00	.00000000E+00
-.52829255E-01	.00000000E+00

EIGENVALUE 10 IS:

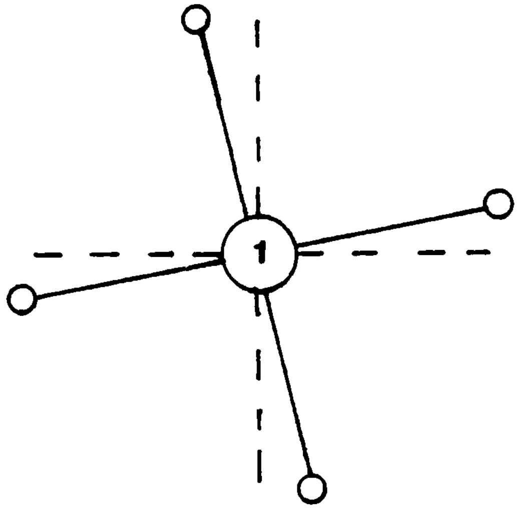
.33367618E+06
.00000000E+00

ITS FREQUENCY IS IN RAD/SEC

.57764711E+03
.00000000E+00

AND ITS EIGENVECTOR IS:

.15361115E-09	.00000000E+00
-.99999994E+00	.00000000E+00
.12120368E+00	.00000000E+00
-.43254721E+00	.00000000E+00
.27977791E+00	.00000000E+00
.41077886E+00	.00000000E+00
.10000000E+01	.00000000E+00
-.12120369E+00	.00000000E+00
.43254724E+00	.00000000E+00
-.27977793E+00	.00000000E+00
-.41077889E+00	.00000000E+00



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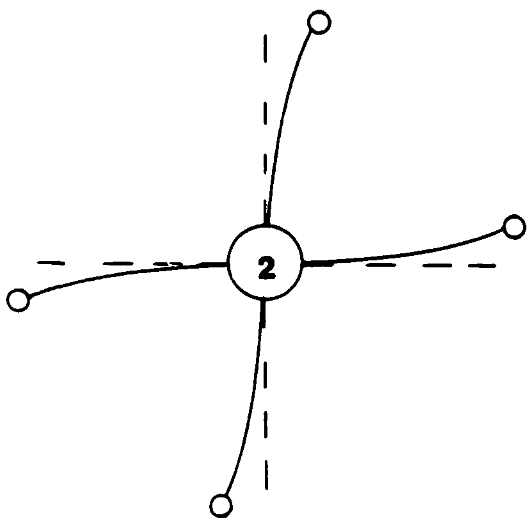

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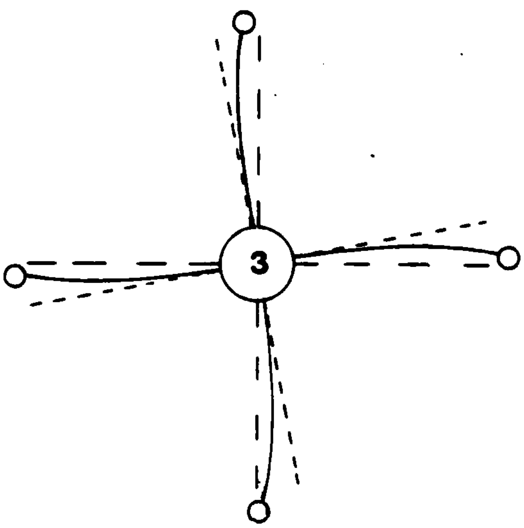
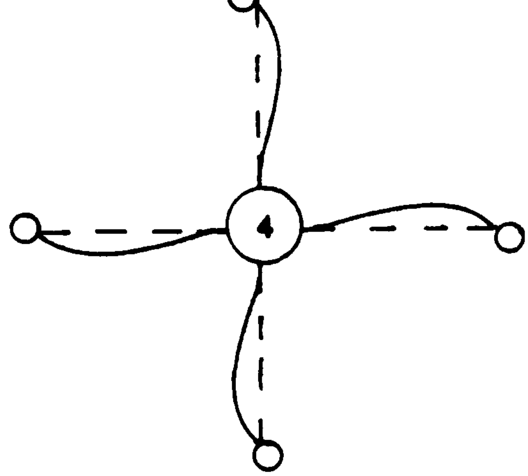
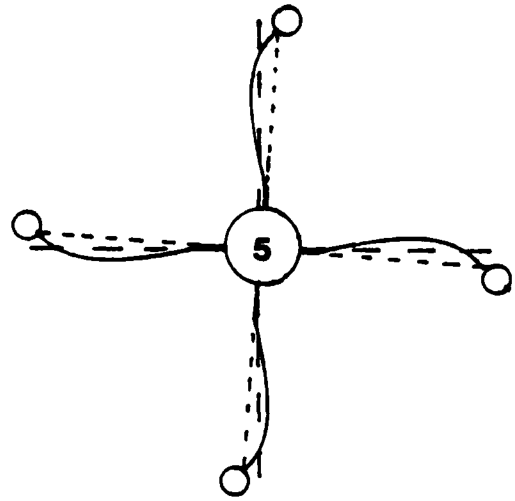


7.91 Rad/Sec



51.45 Rad/Sec



52.81 Rad/Sec

APPENDIX B

EIGENVALUES AND EIGENVECTORS OF CONTROLLED SYSTEM

The eigenvalues and left and right eigenvectors were calculated by using the gains found optimum by Junkins in [4] and the development presented in Chapter 2 for the controlled system. The first 10 eigenvalues and corresponding eigenvectors were calculated and found to all be stable (in the left half plane). The computer program used to perform the calculations is described in Appendix C and listed in subroutine form as MEIGLR. The optimum control law developed by Junkins resulted in low frequencies in the stable left half plane.

EIGENVALUE 1 IS:

-.90228088E-01
-.31614593E+01

AND ITS EIGENVECTOR IS:

.68011813E-12	-.67319834E-12
-.17276375E-03	.18962959E-01
-.70356592E-04	.68196927E-02
.17276372E-03	-.18962959E-01
.70356580E-04	-.68196927E-02
-.38191026E-10	-.36293787E-10
.10000000E+01	.00000000E+00
.35963626E+00	.43371307E-03
-.10000000E+01	.18831664E-08
-.35963626E+00	-.43371241E-03

ITS LEFT EIGENVECTOR IS :

-.17068115E+00	.14464482E+00
.10000000E+01	-.45882435E-12
.26204397E+00	.12731568E+00
.10000000E+01	.00000000E+00
.26204397E+00	.12731568E+00
-.56003427E-01	-.32118343E-01
.12102168E+00	.60673225E-01
-.95076868E-03	-.16860407E-03
.12102168E+00	.60673225E-01
-.95076868E-03	-.16860407E-03

EIGENVALUE 2 IS:

-.90228088E-01
.31614593E+01

AND ITS EIGENVECTOR IS:

.68011813E-12	.67319834E-12
-.17276375E-03	-.18962959E-01
-.70356592E-04	-.68196927E-02
.17276372E-03	.18962959E-01
.70356580E-04	.68196927E-02
-.38191026E-10	.36293787E-10
.10000000E+01	.00000000E+00
.35963626E+00	-.43371307E-03
-.10000000E+01	-.18831664E-08
-.35963626E+00	.43371241E-03

ITS LEFT EIGENVECTOR IS :

-.17068115E+00	-.14464482E+00
.10000000E+01	.45882435E-12
.26204397E+00	-.12731568E+00
.10000000E+01	.00000000E+00
.26204397E+00	-.12731568E+00
-.56003427E-01	.32118343E-01
.12102168E+00	-.60673225E-01
-.95076868E-03	.16860407E-03
.12102168E+00	-.60673225E-01
-.95076868E-03	.16860407E-03

EIGENVALUE 3 IS:

-.13384554E+00
-.43876209E+01

AND ITS EIGENVECTOR IS:

.40064767E-11	-.37278105E-11
-.69461113E-02	.22770205E+00
-.47790885E-03	-.30178248E-02
.69461113E-02	-.22770205E+00
.47790885E-03	.30178248E-02
-.19078769E-10	-.16823943E-10
.10000000E+01	.10981794E-14
-.13177105E-01	.25008053E-02
-.10000000E+01	.32901455E-11
.13177105E-01	-.25008053E-02

ITS LEFT EIGENVECTOR IS :

-.84010789E-02	.91116440E-02
-.94582795E+00	-.47319578E-02
-.26137172E+00	-.14350268E+00
.10000000E+01	.00000000E+00
.27293433E+00	.15131088E+00
-.22747278E-02	-.16012045E-02
-.87306235E-01	-.46890308E-01
.13832321E-02	.18880546E-03
.93596451E-01	.51116625E-01
-.14551629E-02	-.21357217E-03

EIGENVALUE 4 IS:

-.13384554E+00
.43876209E+01

AND ITS EIGENVECTOR IS:

.40064767E-11	.37278105E-11
-.69461113E-02	-.22770205E+00
-.47790885E-03	.30178248E-02
.69461113E-02	.22770205E+00
.47790885E-03	-.30178248E-02
-.19078769E-10	.16823943E-10
.10000000E+01	-.10981794E-14
-.13177105E-01	-.25008053E-02
-.10000000E+01	-.32901455E-11
.13177105E-01	.25008053E-02

ITS LEFT EIGENVECTOR IS :

-.84010789E-02	-.91116440E-02
-.94582795E+00	.47319578E-02
-.26137172E+00	.14350268E+00
.10000000E+01	.00000000E+00
.27293433E+00	-.15131088E+00
-.22747278E-02	.16012045E-02
-.87306235E-01	.46890308E-01
.13832321E-02	-.18880546E-03
.93596451E-01	-.51116625E-01
-.14551629E-02	.21357217E-03

EIGENVALUE 5 IS:

-.15949736E+00
-.47561360E+01

AND ITS EIGENVECTOR IS:

-.90201333E-02	.31605219E+00
-.21421430E-02	-.10770538E+00
-.24183730E-04	-.46681716E-02
-.20928402E-02	-.10849342E+00
-.23056809E-04	-.467241133E-02
.10000000E+01	-.14370798E-14
-.34031291E+00	.16490349E-01
-.14756053E-01	.49765608E-03
-.34280870E+00	.16405583E-01
-.14769564E-01	.494447608E-03

ITS LEFT EIGENVECTOR IS :

-.39933862E+00	.34835071E+00
.10000000E+01	-.48223966E-12
.17964978E+00	.16696430E+00
.10000000E+01	.17729399E-14
.17964978E+00	.16696430E+00

-.80512903E-01	-.75527617E-01
.10246211E+00	.93405672E-01
-.13506535E-02	-.71085598E-03
.10246211E+00	.93405672E-01
-.13506535E-02	-.71085598E-03

EIGENVALUE 6 IS:

-.15949736E+00
.47561360E+01

AND ITS EIGENVECTOR IS:

-.90201333E-02	-.31605219E+00
-.21421430E-02	.10770538E+00
-.24183730E-04	.46681716E-02
-.20928402E-02	.10849342E+00
-.23056809E-04	.46724133E-02
.10000000E+01	.14370798E-14
-.34031291E+00	-.16490349E-01
-.14756053E-01	-.49765608E-03
-.34280870E+00	-.16405583E-01
-.14769564E-01	-.49447608E-03

ITS LEFT EIGENVECTOR IS :

-.39933862E+00	-.34835071E+00
.10000000E+01	.48223966E-12
.17964978E+00	-.16696430E+00
.10000000E+01	-.17729399E-14
.17964978E+00	-.16696430E+00
-.80512903E-01	.75527617E-01
.10246211E+00	-.93405672E-01
-.13506535E-02	.71085598E-03
.10246211E+00	-.93405672E-01
-.13506535E-02	.71085598E-03

EIGENVALUE 7 IS:

-.48040152E+00
.52730010E+02

AND ITS EIGENVECTOR IS:

-.70429861E-02	-.21001852E+00
-.61138169E-03	.13157677E+00
.17135024E-03	.39223396E-02
-.76701797E-03	.13024501E+00
.17005318E-03	.39501266E-02
.10000000E+01	.30518120E-14
-.62569949E+00	-.23893961E-01

-.18682511E-01	.18936222E-03
-.61934063E+00	-.24421776E-01
-.18814462E-01	.17876127E-03

ITS LEFT EIGENVECTOR IS :

-.15044437E-06	-.43499205E-07
-.99988239E+00	.28759423E-05
-.35959628E+00	.25707392E-05
.10000000E+01	-.80986904E-15
.35963852E+00	-.15594396E-05
-.27168420E-08	.27918120E-08
-.18395356E-03	.18958907E-01
-.61921343E-04	.68183756E-02
.18403100E-03	-.18961137E-01
.61947106E-04	-.68191764E-02

EIGENVALUE 8 IS:

-.48040152E+00
-.52730010E+02

AND ITS EIGENVECTOR IS:

-.70429861E-02	.21001852E+00
-.61138169E-03	-.13157677E+00
.17135024E-03	-.39223396E-02
-.76701797E-03	-.13024501E+00
.17005318E-03	-.39501266E-02
.10000000E+01	-.30518120E-14
-.62569949E+00	.23893961E-01
-.18682511E-01	-.18936222E-03
-.61934063E+00	.24421776E-01
-.18814462E-01	-.17876127E-03

ITS LEFT EIGENVECTOR IS :

-.15044437E-06	.43499205E-07
-.99988239E+00	-.28759423E-05
-.35959628E+00	-.25707392E-05
.10000000E+01	.80986904E-15
.35963852E+00	.15594396E-05
-.27168420E-08	-.27918120E-08
-.18395356E-03	-.18958907E-01
-.61921343E-04	-.68183756E-02
.18403100E-03	.18961137E-01
.61947106E-04	.68191764E-02

EIGENVALUE 9 IS:

-.49446867E+00
.53917505E+02

AND ITS EIGENVECTOR IS:

-.13121624E-04	.73481477E-03
-.17007586E-03	-.18545293E-01
-.71610986E-04	-.64743029E-02
-.17010004E-03	-.18543276E-01
-.71618821E-04	-.64735798E-02
-.39612890E-01	-.10708281E-02
.10000000E+01	.69625827E-15
.34911367E+00	-.65974577E-03
.99989129E+00	-.23008404E-05
.34907468E+00	-.66052574E-03

ITS LEFT EIGENVECTOR IS :

-.36828520E-01	-.72612925E-05
.10000000E+01	-.10866528E-11
.34997737E+00	-.37209655E-05
.10000000E+01	.00000000E+00
.34997737E+00	-.37209651E-05
-.27042912E-04	.68269453E-03
.19999138E-03	-.18542003E-01
.59158834E-04	-.64893800E-02
.19999138E-03	-.18542003E-01
.59158834E-04	-.64893800E-02

EIGENVALUE 10 IS:

-.49446867E+00
-.53917505E+02

AND ITS EIGENVECTOR IS:

-.13121624E-04	-.73481477E-03
-.17007586E-03	.18545293E-01
-.71610986E-04	.64743029E-02
-.17010004E-03	.18543276E-01
-.71618821E-04	.64735798E-02
-.39612890E-01	.10708281E-02
.10000000E+01	-.69625827E-15
.34911367E+00	.65974577E-03
.99989129E+00	.23008404E-05
.34907468E+00	.66052574E-03

ITS LEFT EIGENVECTOR IS :

-.36828520E-01	.72612925E-05
.10000000E+01	.10866528E-11
.34997737E+00	.37209655E-05
.10000000E+01	.00000000E+00
.34997737E+00	.37209651E-05
-.27042912E-04	-.68269453E-03
.19999138E-03	.18542003E-01
.59158834E-04	.64893800E-02
.19999138E-03	.18542003E-01
.59158834E-04	.64893800E-02

APPENDIX C

COMPUTER PROGRAMS

The following computer program contains the programs used in this report in subroutine form,except for subroutine CONMIN which can be obtained through NASA or Dr.Khot, WPAFB.

Program MASMIN initializes all parameters required by CONMIN and dimensions the arrays common to CONMIN. It prompts the user for values for DELFUN, DABFUN, CT, and THETA and then asks for the initial values for t, h, and L. The lower and upper bounds on the decision variables are set and CONMIN called to begin the process. Subroutine CALOBJ is called to calculate the objective function, gradients of the objective function, constraints and gradients of the constraints. File MINMAS is passed the final values of the objective function and decision variables.

Subroutine CALOBJ calculates the gradients of the constraints, which were ω and ξ of a specified eigenvalue, by calling subroutine PHICAL. Matrix A, which is common to CONMIN, contains the gradients of the constraints while array DF contains the gradients of the objective function.

Subroutine PHICAL calculates the mass and stiffness matrices for specified values of t, h, and L. This data is stored in files 'MMASS' and 'MSTIFF' for separate access. PHICAL calls subroutine CALC1 which computes values for $\phi(z)$, $\phi'(z)$, integral of $\phi_i\ \phi_j$, integral of $x\phi_i(z)$, and the integral of $\phi'_i\phi'_j$ numerically. These values may be printed

out by the user if desired and are used to form the mass and stiffness matrices.

Subroutine MEIGLR is passed the mass and stiffness matrices, the length of the appendages, and the number of modes being examined by PHICAL and it uses this data to form matrices A and B. An IMSL subroutine called EIGZF is passed matrices A and B which solves for the eigenvalues and eigenvectors. The transpose of A and B are also passed to EIGZF to return the left eigenvectors. MEIGLR forms the A and B matrices by calculating the B(called BZ in the program) and H matrices as shown in Chapter 2 and calling subroutine Gain which contains the gain values. The matrices are assembled with the previously calculated mass and stiffness matrices. Matrix A and B may also be printed if desired along with the eigenvalues and eigenvectors.

Subroutine CALCBA is passed the specified or input values for t, h, L, the number of modes being examined, the values of the integrals calculated by PHICAL along with the eigenvalues and eigenvectors. It outputs the gradients of ω and ξ for the specified eigenvalue. In other words if the third eigenvalue is being used as a constraint, then the user should output the gradients of ω_3 and ξ_3. The gradients of the mass and stiffness matrices along with Bz and H are computed for each structural parameter and used to form the gradients of the A and B matrices.

The gradients of A and B are passed to subroutine

GRADNT which forms the gradients of the eigenvalues. This is accomplished in accordance with the development in Chapter 3 using the partial derivatives of A and B (Chapter 3 refers to matrix B as D to avoid confusion with the control matrix B). Once the gradients of the eigenvalues are found, the gradients of ω and ξ are easily formed and are referred to as PW3 and PZ3. These represent 3 element arrays containing the partial derivative of ω_3 w.r.t. t, h, and L and the partial derivative of ξ_3 w.r.t. t, h, and L, respectively.

This information is passed back to subroutine CALOBJ which aids CONMIN in its constrained minimization problem. The gradients of ω_3 and ξ_3 along with those of the objective function provide direction for the method of Feasible Directions used by CONMIN.

```
      PROGRAM MASMIN
      COMMON/CNMN1/DELFUN,DABFUN,FDCH,FDCHM,CT,CTMIN,CTL,CTLMIN,
     1ALPHAX,ABOBJ1,THETA,OBJ,NDV,NCON,NSIDE,IPRINT,NFDG,
     2NSCAL,LINOBJ,ITMAX,ITRM,ICNDIR,IGOTO,NAC,INFO,INFOG,ITER
      COMMON/CONSAV/REAL(50),INT(25)
      DIMENSION X(50),VLB(50),VUB(50),G(1100),SCAL(50),DF(50),
     1A(5,51),S(51),G1(1100),G2(1100),B(51,51),C(51),
     2ISC(1000),IC(51),MS1(102)
C
C       DELFUN  AND DABFUN SPECIFY THE TOLERANCE ON THE  OBJECTIVE
C   FUNCTION,   CT   IS   THE   CONSTRAINT  THICKNESS  PARAMETER   AND
C SPECIFIES  THE TOLERANCE ON SATISFYING THE CONSTRAINTS.    THETA
C   IS THE PUSH  OFF FACTOR AND IPRINT ALLOWS THE USER TO  SPECIFY
C HOW MUCH OUTPUT  HE WANTS FROM CONMIN.


      PRINT*,'PLEASE INPUT DELFUN,DABFUN,CT,THETA,IPRINT'
      READ*,DELFUN,DABFUN,CT,THETA,IPRINT
      PRINT*,'PLEASE INPUT THE VALUES FOR T,H,L'
      DO 15 I=1,3
        READ*,X(I)
15        CONTINUE
C
      FDCH = 0.0
      FDCHM = 0.0
      CTMIN = 0.0
      CTL = 0.0
      CTLMIN = 0.0
      ALPHAX = 0.1
      ABOBJ1 = 0.1
      NDV = 3
      NCON = 2
      NFDG = 1
      NSCAL = 0
      LINOBJ = 0
      ITMAX = 40
      ITRM = 0
      ICNDIR = 0
      NSIDE = 6
      IGOTO = 0
      N1 = 5
      N2 = 8
      N3 = 51
      N4 = 51
      N5 = 102
C
      DO 10 I = 1,NCON

      ISC(I) = 0
   10 CONTINUE
      DO 20 I = 1,NDV
         VLB(I)=.000001
         VUB(I)=1.0E+10
```

```
   20 CONTINUE
C
C
      OPEN(UNIT=9,FILE='MINMAS',ACCESS='SEQUENTIAL',STATUS='NEW')
       REWIND(UNIT=9)
   50 CALL CONMIN(X,VLB,VUB,G,SCAL,DF,A,S,G1,G2,B,C,ISC,
     1            IC,MS1,N1,N2,N3,N4,N5)
      IF (IGOTO .EQ. 0) GO TO 200
      IF (INFO .EQ. 2) GO TO 150
      CALL CALOBJ(X,G,DF,A,IC,N3)
      GO TO 50
  150 CALL CALOBJ(X,G,DF,A,IC,N3)
      GO TO 50
  200 CALL CALOBJ(X,G,DF,A,IC,N3)
      WRITE(9,185) X(1),X(2),X(3)
  185 FORMAT(5X,7HX(1) = ,E15.8,5X,7HX(2) = ,E15.8,5X,
     17HX(3) = ,E15.8/)
      WRITE(9,187)OBJ,G(1),G(2)
  187 FORMAT(5X,21HOBJECTIVE FUNCTION = ,E15.8,/5X,
     121HCONSTRAINT 1       = ,E15.8,/5X,
     221HCONSTRAINT 2       = ,E15.8/)
        ENDFILE(UNIT=9)
 1000 STOP

      END
     ***************************************************************

     ***************************************************************
       SUBROUTINE CALOBJ(X,G,DF,A,IC,N3)
      COMMON/CNMN1/DELFUN,DABFUN,FDCH,FDCHM,CT,CTMIN,CTL,CTLMIN,
     1ALPHAX,ABOBJ1,THETA,OBJ,NDV,NCON,NSIDE,IPRINT,NFDG,
     2NSCAL,LINOBJ,ITMAX,ITRM,ICNDIR,IGOTO,NAC,INFO,INFOG,ITER
      DIMENSION X(50),G(1100),DF(50),IC(51),A(5,51)
C
       INTEGER I
        COMPLEX LBDA(10)
        REAL MD,T,H,L,PW3(3),PZ3(3),W3,Z3,Z
******************************************************
C
C
C  THIS CALCULATES THE OBJECTIVE FUNCTION AND GRADIENTS
C
C
200   FORMAT(5X,'T = ',E15.8,5X,'H = ',E15.8,5X,'L = ',E15.8/)
C
      T=X(1)
      H=X(2)
      L=X(3)
C
        MD=5.22
        CALL PHICAL(T,H,L,PW3,PZ3,LBDA)
C INPUT IS T,H,L
```

```
      IF (INFO.EQ.2)GO TO 10
C
C  CALCULATE OBJECTIVE FUNCTION AND GRADIENTS
C
    OBJ=4*MD*T*H*L
      WRITE(*,*)'THIS IS ITERATION   ',ITER
      WRITE(*,*)'THIS IS THE OBJECTIVE FUNCTION  ',OBJ
     WRITE(*,*)'THIS IS LAMBDA          ',LBDA(3)
     WRITE(*,200) X(1),X(2),X(3)
C
C    CALCULATE GRADIENTS G(1) AND G(2)
C    W3 IS RE-COMPUTED VALUE OF OMEGA
C    Z3 IS RE-COMPUTED VALUE OF ZETA, FOR THIRD EIGENVALUE
C
      W3 =SQRT((REAL(LBDA(3)))**2 + (AIMAG(LBDA(3)))**2)
       Z=REAL(LBDA(3))
       Z3=-Z/W3
       G(1)=W3-4.6000
       G(2)= 4.2-W3
C
      IF (INFO.EQ.1) RETURN
10    CONTINUE
C
    DF(1)=4*MD*H*L
    DF(2)=4*MD*T*L
    DF(3)=4*MD*T*H
C
     NAC=0
     IF (G(1).LT.CT) GO TO 20
     NAC=1
      IC(1)=1
    A(1,1)=-PW3(1)
    A(2,1)=-PW3(2)
    A(3,1)=-PW3(3)
C
20    IF (G(2).LT.CT) RETURN
    NAC=NAC+1
    IF (NAC.EQ.N3) RETURN
       IC(NAC)=2
    A(1,NAC)=PW3(1)
    A(2,NAC)=PW3(2)
    A(3,NAC)=PW3(3)
C
C
      END
C
*****************************************************************
```

```
  SUBROUTINE PHICAL(T,H,L,PW3,PZ3,LBDA)
      INTEGER I,J,N,K
        REAL IX,PW3(3),PZ3(3)
      REAL PHI(10),PHISQ(10,10),IPHISQ(10,10),PHI1(10),PHI1SQ(10,10)
      REAL IXPHI(10),TEMP1A(10,10),TEMP1B(10,10),TEMP2A(10),TEMP2B(10)
      REAL M(21,21),C1,C,I2C,I1,M2,D,T,H,AX,E
        REAL HZ(5,5)
        COMPLEX LIGVCT(10,10),RIGVCT(10,10),LBDA(10)
        REAL STIFF(21,21)
C        C IS THE MOMENT OF INERTIA OF THE HUB
C        I2C IS THE ROTARY MOMENT OF INERTIA OF THE PT MASS
C        D IS THE DENSITY OF THE ARMS
C        I1 IS THE TRANSLATED MOMENT OF INERTIA OF THE ARMS
C         T IS THE THICKNESS OF THE ARMS
C         H IS THE HEIGHT OF THE ARMS
C         L IS THE LENGTH OF THE ARMS
C        E IS THE MODULUS OF ELASTICITY OF THE ARMS
C        IX IS THE AREA MOMENT OF INERTIA OF THE ARMS
        REAL R, X,L,IPHI2(10,10),TEMP3A(10,10),TEMP3B(10,10)
         N=2
         R=1
         D=5.22
         M2=.156941
         I2C=.0018
         E=1.584E9
        X=R
      CALL CALC1(X,L,N,PHI,PHISQ,TEMP1A,PHI1,PHI1SQ,TEMP2A,TEMP3A)
      X=R+L
      CALL CALC1(X,L,N,PHI,PHISQ,TEMP1B,PHI1,PHI1SQ,TEMP2B,TEMP3B)
      DO 40 I=1,N
        DO 50 J=1,N
          IPHISQ(I,J)=TEMP1B(I,J)-TEMP1A(I,J)
          IXPHI(I)=TEMP2B(I)-TEMP2A(I)
        IPHI2(I,J)=TEMP3B(I,J)-TEMP3A(I,J)
 50     CONTINUE
40      CONTINUE

C     TO PRINT THE PHI,PHISQ,IPHISQ ETC. MATRICES THEN REMOVE THE
C      IF STATEMENT.

          IF(I.EQ.111)THEN

      PRINT*,'THE FOLLOWING IS PHI'
      PRINT 100,(PHI(I),I=1,N)
      PRINT*,'THE FOLLOWING IS PHISQ'
        DO 60 I=1,N
      PRINT  100,(PHISQ(I,J),J=1,N)
 60       CONTINUE
      PRINT*,'THIS IS IPHISQ'
        DO 70 I=1,N
      PRINT 100,(IPHISQ(I,J),J=1,N)
 70      CONTINUE
      PRINT*,'THIS IS PHI1'
```

```
      PRINT 100,(PHI1(I),I=1,N)
      PRINT*,'THIS IS PHI1SQ'
        DO 80 I=1,N
      PRINT 100,(PHI1SQ(I,J),J=1,N)
80          CONTINUE
      PRINT*,'THIS IS IXPHI'
      PRINT 100,(IXPHI(I),I=1,N)
        PRINT*,'THE FOLLOWING RESULTS ARE IPHI2'
        PRINT*,' '
        DO 85 I=1,N
        PRINT 100,(IPHI2(I,J),J=1,N)
85      CONTINUE
          ENDIF

      C=8.0
      AX=D*T*H
      I1=(AX/3)*((R+L)**3-R**3)
      C1=(C/2)+2*I1+2*M2*(R+L)**2
        IX=(1/12.0)*H*T**3
      M(1,1)=2*(C1+I2C)
       STIFF(1,1)=0.0

      DO 110 I=1,N
        M(1,I+1)=2*(AX*IXPHI(I)+(R+L)*M2*PHI(I)+I2C*PHI1(I))
        M(1,I+N+1)=M(1,I+1)
        M(I+1,1)=M(1,I+1)
        M(I+N+1,1)=M(1,I+1)
          STIFF(1,I+1)=0.0
          STIFF(1,I+N+1)=0.0
          STIFF(I+1,1)=0.0
          STIFF(I+N+1,1)=0.0
 110        CONTINUE

      DO 120 I=1,N
        DO 130 J=1,N
           M(I+1,J+1)=2*(AX*IPHISQ(I,J)+M2*PHISQ(I,J)+I2C*PHI1SQ(I,J))
           M(I+N+1,J+N+1)=M(I+1,J+1)
           M(I+1,J+N+1)=0
           M(I+N+1,J+1)=0
           STIFF(I+1,J+1)=2*(E*IX*IPHI2(I,J))
         STIFF(I+1+N,J+1)=0.0
         STIFF(I+1,J+1+N)=0.0
           STIFF(I+1+N,J+1+N)=2*(E*IX*IPHI2(I,J))
 130       CONTINUE

  120      CONTINUE

100        FORMAT(8(E16.8))
        CALL MEIGLR(L,N,M,STIFF,LIGVCT,RIGVCT,LBDA)
       CALL CALCBA(T,H,L,N,IXPHI,IPHISQ,IPHI2,LIGVCT,
     1   RIGVCT,LBDA,PW3,PZ3)
        END
```

```
*        ************************************************
         SUBROUTINE CALC1(Z,L,N,TEMP1,TEMP2,MP,TEMP4,TEMP5,TEMP6
1        REAL X,L,TEMP1(TEMP7EMP2(10,10),   MP(10,10),TEMP4(10)
          REAL MP1,MP2,MP3,MP4,MP5,Z,R
        INTEGER I,J,N
        REAL TEMP5(10,10),TEMP6(10),A,B,C,D,F,TEMP7(10,10)
          R=1.0
          X=Z-R
        PI=3.141593
        DO 10 I=1,N
          DO 20 J=1,N
            A=I*PI/L
            B=J*PI/L
            C=(-1)**(I+1)
            D=(-1)**(J+1)
           IF (I.NE.J) THEN
               F=(1/(2*(A-B)))*SIN((A-B)*X)+(1/(2*(A+B)))
     1        *SIN((A+B)*X)
          ELSE
               F=.5*X +(1/(4*A))*SIN(2*A*X)
            ENDIF
          TEMP1(I)=1-COS(A*X)+.5*C*(A*X)**2
          TEMP2(I,J)=1-COS(B*X)+.5*D*(B*X)**2 -COS(A*X)+COS(A*X)
     1     *COS(B*X)-
     1      .5*D*(B*X)**2*COS(A*X)+.5*C*(A*X)**2-.5*C*(A*X)**2
     1     *COS(B*X)
     1      +.25*C*D*(A*B)**2*X**4
          MP1=X-(1/B)*SIN(B*X)+(1/6.0)*D*B*B*X**3-(1/A)
     1     *SIN(A*X)
          MP2=-.5*D*B*B*
     1      ((1/A**2)*2*X*COS(A*X)+(1/A**3)*((A*X)**2-2)
     1     *SIN(A*X))
           MP3=(1/6.0)*C*A*A*X**3
           MP4=-.5*C*A*A*((1/B**2)*2*X*COS(B*X)+(1/B**3)
     1     *((B*X)**2-2)*SIN(B*X))
           MP5=(1/20.0)*C*D*A*A*B*B*X**5 + F
          MP(I,J)=MP1+MP2+MP3+MP4+MP5
         TEMP4(I)=A*SIN(A*X)+C*A*A*X
         TEMP5(I,J)=A*B*SIN(A*X)*SIN(B*X)+A*D*B*B*X*SIN(A*X)
     1     +A*A*B*C*X*SIN(B*X)+A*A*B*B*C*D*X*X

          TEMP6(I)=.5*Z*Z-(1/A**2)*COS(A*X)-(Z/A)*SIN(A*X)
     1    +.5*C*(A**2)*(.25*(Z**4)-(2/3.0)*R*Z**3+.5*R*R*Z*Z)
          TEMP7(I,J)=((A*B)**2)*(F+(C/B)*SIN(B*X)+(D/A)*SIN(A*X)
     1      +C*D*X)
  20      CONTINUE
   10     CONTINUE

           END


          SUBROUTINE MEIGLR(L,P,M,S,LIGVCT,Z,EIGENV)
```

```
C ******************************************************************
C ******************************************************************
C
C      THIS PROGRAM SOLVES FOR THE EIGENVALUES AND EIGENVECTORS BY
C     SOLVING THE PROBLEM AX = LAMBDA BX.   A IS FORMED  FROM  THE
C      STIFFNESS AND THE CONTROL MATRICES,  WHILE B IS FORMED FROM
C     THE MASS MATRIX.
C
C     Z CONTAINS THE RIGHT EIGENVECTOR ,  LIGVCT CONTAINS THE LEFT
C       EIGENVECTOR, AND EIGENV IS THE EIGENVALUES.
C
       REAL A(10,10),B(10,10),BETA(10),WK(888),S(21,21),M(21,21)
         REAL G1(3,5),G2(3,5),SUM1,SUM2,TEMP1,TEMP2
         REAL BZG1(21,5),BZG2(21,5),BZG1H(21,21),BZG2H(21,21)
         REAL BZ(21,3),H(5,5),Z1,Z2,Z3,Z4,R,L,Y,PHI(21),PHI1(21)
       REAL COMP1,COMP2
       COMPLEX ALFA(10),Z(10,10),EIGENV(10),OMEGA(21),TEMP(21)
       COMPLEX VECT(21,21),LIGVCT(10,10),VECTL(10,10)
       INTEGER I,J,IA,IB,N,IJOB,IZ,IER,Q,X,K,P
          INTEGER P1,Q2
       REAL ZZ(21,21),ATRANS(10,10),BTRANS(10,10)
C
5          FORMAT(E16.8)
6        FORMAT(8(E16.8))
        Q=2*P+1
         Q2=2*Q
C       NOTE Q IS THE DIMENSION OF THE INPUT MASS AND STIFFNESS
C       MATRIX AND Q2 IS      THE DIMENSION OF THE OUTPUT EIGENVECTOR
C       AND FREQUENCY MATRIX.

        BZ(1,1)=1.0
        BZ(1,2)=2.0
        BZ(1,3)=2.0
        Z1=L/2.0
        Z2=L/2.0
        Z3=L/2.0
        Z4=L
          R=Z1

Y=Z3
         CALL CALC(R,Y,L,P,PHI,PHI1)
           P1=P+1
        DO 21 I=1,P
          H(2,I+1)=PHI(I)
          H(4,I+P1)=PHI(I)
          BZ(I+1,2)=2*PHI1(I)
          BZ(I+P1,1)=0.0
          BZ(I+P1,2)=0.0
          BZ(I+1,1)=0.0
          BZ(I+1,3)=0.0
          H(1,I+1)=0.0
          H(1,I+P1)=0.0
          H(2,I+P1)=0.0
```

```
        H(3,I+P1)=0.0
        H(4,I+1)=0.0
        H(5,I+1)=0.0
21      CONTINUE
C
       H(1,1)=1.0
       H(2,1)=0.0
       H(3,1)=0.0
       H(4,1)=0.0
       H(5,1)=0.0
      R=Z2
      Y=Z4
        CALL CALC(R,Y,L,P,PHI,PHI1)
      DO 30 I=1,P
        H(3,I+1)=PHI(I)
        H(5,I+P1)=PHI(I)
        BZ(I+P1,3)=2*PHI1(I)
30      CONTINUE
C     READ IN G1 AND G2
C
       CALL GAIN(G1,G2)
C
C    MULTIPLY BZ * G1 AND BZ * G2
C
      DO 50 I=1,Q
        DO 60 J=1,5
          SUM1=0.0
          SUM2=0.0
           DO 70 K=1,3
              TEMP1=BZ(I,K)*G1(K,J)
              TEMP2=BZ(I,K)*G2(K,J)

              SUM1=SUM1+TEMP1
              SUM2=SUM2+TEMP2
70      CONTINUE
               BZG1(I,J)=SUM1
               BZG2(I,J)=SUM2
60     CONTINUE
50     CONTINUE
C     MULTIPLY BZG1*H  AND BZG2*H
C   THIS ASSUMES H1=H2
      DO 80 I=1,Q
         DO 90 J=1,Q
           SUM1=0.0
           SUM2=0.0
            DO 95 K=1,5
               TEMP1=BZG1(I,K)*H(K,J)
               TEMP2=BZG2(I,K)*H(K,J)
               SUM1=SUM1+TEMP1
               SUM2=SUM2+TEMP2
                BZG1H(I,J)=SUM1
                BZG2H(I,J)=SUM2
95      CONTINUE
```

```
90        CONTINUE
80        CONTINUE
C
C  FORM A AND B MATRICES
C
         DO 100 I=1,Q
           DO 110 J=1,Q
             A(I,J)=0.0
             A(I,J+Q)=-M(I,J)
             A(I+Q,J)=(S(I,J)+BZG1H(I,J))
             A(I+Q,J+Q)=BZG2H(I,J)
             B(I,J)=-M(I,J)
             B(I+Q,J+Q)=-M(I,J)
             B(I+Q,J)=0.0
             B(I,J+Q)=0.0
110        CONTINUE
100       CONTINUE
C
C   SAVE A MATRIX IN FILE AMAT AND B IN FILE BMAT
C      TO PRINT OR SAVE REMOVE THE IF STATEMENT
C
           IF(I.EQ.111)THEN

C CHECK THAT MATRICES WERE READ IN PROPERLY
          PRINT*,'THIS IS THE A MATRIX'
        DO 500 I=1,Q
            WRITE(*,6)(A(I,J),J=1,Q)
500     CONTINUE
        PRINT*,' '
        PRINT*,'THE B MATRIX IS:'
        PRINT*,' '
        DO 520 I=1,Q
            WRITE(*,6)(B(I,J),J=1,Q)
520     CONTINUE
C
           ENDIF


          IA=10
          IB=10
          IZ=10
          IJOB=2
          N=Q2
C
        CALL EIGZF (A,IA,B,IB,N,IJOB,ALFA,BETA,Z,IZ,WK,IER)
C
        DO 300 I=1,10
          DO 310 J=1,10
            ATRANS(I,J)=A(J,I)
            BTRANS(I,J)=B(J,I)
310       CONTINUE
300       CONTINUE
        CALL EIGZF(ATRANS,IA,BTRANS,IB,N,IJOB,ALFA,BETA,
```

```
     1  LIGVCT,IZ,WK,IER)
        Q=10
C
       DO 220 I=1,Q
          IF (BETA(I).NE.0) THEN
          EIGENV(I)=ALFA(I)/BETA(I)
          ELSE
        EIGENV(I)=999999.9
          ENDIF
220    CONTINUE
C
C SORT THE EIGENVALUES AND EIGENVECTORS
C
       X=Q-1
221    IF (X.NE.0) THEN
DO 222 I=1,X
          K=I+1
        COMP1=REAL(EIGENV(K))
        COMP2=REAL(EIGENV(I))
          IF (COMP1.GT.COMP2) THEN
            TEMP(I)=EIGENV(I)
            EIGENV(I)=EIGENV(K)
            EIGENV(K)=TEMP(I)
            DO 223 J=1,Q
          VECTL(J,I)=LIGVCT(J,I)
          LIGVCT(J,I)=LIGVCT(J,K)
          LIGVCT(J,K)=VECTL(J,I)
              VECT(J,I)=Z(J,I)
              Z(J,I)=Z(J,K)
              Z(J,K)=VECT(J,I)
223         CONTINUE
            ENDIF
222      CONTINUE
       X=X-1
       GOTO 221
       ENDIF
       DO 225 I=1,Q
         DO 226 J=1,Q
           ZZ(I,J)=REAL(Z(I,J))
226    CONTINUE
225    CONTINUE
8         FORMAT(2(E16.8))
        IF (I.EQ.111) THEN
       DO 240 I=1,Q
         PRINT*,'EIGENVALUE ',I,' IS:'
         PRINT*,' '
         WRITE(*,5)EIGENV(I)
         PRINT*,' '
         OMEGA(I)=SQRT(EIGENV(I))
         PRINT*,'ITS FREQUENCY IS IN RAD/SEC'
         PRINT*,' '
         WRITE(*,5)OMEGA(I)
         PRINT*,' '
```

```
          PRINT*,'AND ITS EIGENVECTOR IS:'
          PRINT*,' '
          WRITE(*,8)(Z(J,I),J=1,Q)
          PRINT*,' '
      PRINT*,'ITS LEFT EIGENVECTOR IS :'
      WRITE(*,8)(LIGVCT(J,I),J=1,Q)
      PRINT*,'   '
240   CONTINUE
         ENDIF
C
      END
       SUBROUTINE CALC(R,Y,L,P,PHI,PHI1)
       REAL R,Y,L,PHI(21),PHI1(21)
       REAL PI,C,D
       INTEGER I,P
       PI=3.14159265
       DO 311 I=1,P
         C=I*PI/L
         D=(-1)**(I+1)
         PHI(I)=1-COS(C*Y)+.5*D*C*C*Y*Y
         PHI1(I)=C*SIN(C*R)+D*C*C*R
311        CONTINUE
            END
***************************************************************

C  THIS PROGRAM CREATES GAIN1 (G1)  AND GAIN2  (G2)
       SUBROUTINE GAIN(G,G2)
      REAL G1(3,5),G2(3,5),G(3,5)
      INTEGER I,J
C
      G(1,2)=-1.08
      G(1,1)=8.60
      G(1,3)=-3.09
      G(1,4)=-1.08
      G(1,5)=-3.09
      G(2,1)=21.8
      G(2,2)=-1.74
      G(2,3)=-4.94

      G(2,4)=-1.74
      G(2,5)=-4.94
      G(3,1)=21.9
      G(3,2)=-1.74
      G(3,3)=-4.94
      G(3,4)=-1.74
      G(3,5)=-4.94
C
      G2(1,1)=-.238
      G2(1,2)=-.016
      G2(1,3)=-.050
      G2(1,4)=-.016
      G2(1,5)=-.050
      G2(2,1)=.643
```

AD-A152 858 INCORPORATING CONTROL INTO THE OPTIMAL STRUCTURAL DESIGN OF LARGE FLEXIBL..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. 2/2

UNCLASSIFIED T V MUCKENTHALER DEC 84 AFIT/GA/AA/84D-7 F/G 22/2 NL

END

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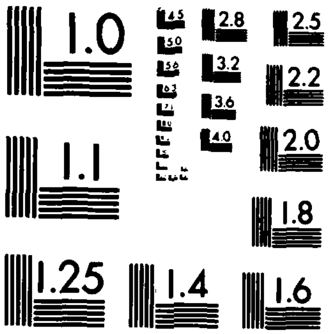


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
      G2(2,2)=.233
      G2(2,3)=.119
      G2(2,4)=-.015
      G2(2,5)=.028
      G2(3,1)=.643
      G2(3,2)=-.015
      G2(3,3)=.028
      G2(3,4)=.233
      G2(3,5)=.119
         END

*****************************************************************
*****************************************************************

        SUBROUTINE CALCBA(T,H,L,P,IXPHI,IPHISQ,IPHI2,LIGVCT,
     1  RIGVCT,LBDA,PW3,PZ3)
        COMPLEX LIGVCT(10,10),RIGVCT(10,10),LBDA(10)
        REAL PW3(3),PZ3(3),PIPHSQ
      REAL P1M(21,211),P2M(21,21),P3M(21,21),P1S(21,21)
      REAL P2S(21,21),P3S(21,21),IXPHI(10),IPHISQ(10,10),IPHI2(10,10)
      REAL MD,X,R,L,T,H,MDIX,MDPH,F,PF,PI,A,B,A2,B2
      REAL C,D,PXPHI,PHI,PHI1,PIPHI2,PHI12,I2C,E
      REAL PBZ(5,3),G1(3,5),PBG1(5,5),HZ(5,5),PBG1H(5,5)
        REAL PHIHF
      REAL G2(3,5),PBG2(5,5),PBG2H(5,5)
      REAL P1A(10,10),P2A(10,10),P3A(10,10),P1B(10,10),P2B(10,10)
      REAL P3B(10,10)
      INTEGER I,J,K,P,KI,KJ
C   THIS PROGRAM ASSUMES BOTH PAIRS OF APPENDAGES ARE KEPT THE SAME
C  P1M(I,J) IS THE PARTIAL DERIVATIVE OF M W.R.T  T
C  P2M(I,J) IS THE PARTIAL DERIVATIVE OF M W.R.T. H
C  P3M(I,J) IS THE PARTIAL DERIVATIVE OF M W.R.T. L
C  MD IS THE MASS DENSITY
      MD=5.22
      E=1.584E+09
      I2C=.0018
      R=1
      M2=.156941
C
      X=R+L
      P1M(1,1)=.6667*MD*H*(X**3-R**3)
      P2M(1,1)=.66667*MD*T*(X**3-R**3)
      P3M(1,1)=2*MD*T*H*X*X + 4*M2*X
C
      DO 110 I=1,P
        DO 120 J=1,P
          MDIX=MD*IXPHI(J)
          MDPH=MD*IPHISQ(I,J)
          P1M(1,J+1)=MDIX*H
          P1M(1,J+1+P)=MDIX*H
          P1M(1+J,1)=MDIX*H
          P1M(1+J+P,1)=MDIX*H
          P1M(I+1,J+1)=MDPH*H
```

```
      P1M(1+I+P,1+J+P)=MDPH*H
      P1M(I+1,J+1+P)=0.0
      P1M(I+1+P,J+1)=0.0
      P2M(1,J+1)=MDIX*T
      P2M(1,J+1+P)=MDIX*T
      P2M(1+J,1)=MDIX*T
      P2M(1+J+P,1)=MDIX*T
      P2M(I+1,J+1)=MDPH*T
      P2M(I+1+P,J+1+P)=MDPH*T
      P2M(I+1+P,J+1)=0.0
      P2M(I+1,J+1+P)=0.0
120   CONTINUE
110   CONTINUE
      PI=3.14159
C
      DO 130 I=1,P
        DO 140 J=1,P
          B=J*PI
         A=I*PI
          A2=A*A
          B2=B*B
          C=(-1)**(I+1)
          D=(-1)**(J+1)

C  PXPHI IS THE PARTIAL DERIVATIVE OF XPHI
          PXPHI=X-2*(L/B2)*COS(B)-D*(B2/L**3)*(.25*X**4-.6667*X**3
     1  +.5*X*X)+.5*D*(B2/L*L)*(X**3-2*X*X+X)+2*(L/B2)+D*(B2/L**3)
     2   *.08333
          PHI=1-COS(B)+.5*D*B2
C     PHIHF IS PHI EVALUATED AT L/2

          PHIHF=1-COS(B/2)+.5*D*.25*B2
C
          HZ(1,1)=1.0
          HZ(1,J+1)=0.0
          HZ(1,J+P+1)=0.0
          HZ(2,1)=0.0
          HZ(2,J+1)=PHIHF
          HZ(2,J+P+1)=0.0
          HZ(3,J+1)=PHI
          HZ(3,J+1+P)=0.0
          HZ(3,1)=0.0
          HZ(4,1)=0.0
          HZ(4,J+1)=0.0
          HZ(4,J+P+1)=PHIHF
          HZ(5,1)=0.0
          HZ(5,J+1)=0.0
          HZ(5,J+P+1)=PHI

C
          PPHI1=-D*B2/(L*L)
          IF (I.NE.J) THEN
             F=(1/(2*(A-B)))*L*SIN((A-B))+(L/(2*(A+B)))
```

```
     1        *SIN((A+B))
              PF=(1/(2*(A-B)))*SIN(A-B)+(1/(2*(A+B)))*SIN(A+B)
            ELSE
             F=.5*L
             PF=.5
           ENDIF
         PPHI12=-2*(A2/L**3)*B2*C*D
C   PPHI12 IS THE PARTIAL OF PHI' SQ W.R.T. L
           PIPHSQ=1+.1667*D*B2-D*(B2/A2)*COS(A)+.16667*C*A2-C*(A2/B2)
     1     *COS(B) +.05*C*D*A2*B2 + PF
C    PIPHSQ IS THE PARTIAL OF THE INTEGRAL OF PHI SQ W.R.T. L
            P3M(1,J+1)=MD*T*H*PXPHI+M2*PHI+I2C*PPHI1
            P3M(1,J+P+1)=P3M(1,J+1)
            P3M(J+1,1)=P3M(1,J+1)
            P3M(J+P+1,1)=P3M(1,J+1)
            P3M(I+1,J+1)=MD*T*H*PIPHSQ+I2C*PPHI12
            P3M(I+P+1,J+P+1)=P3M(I+1,J+1)
            P3M(I+1,J+P+1)=0.0
            P3M(I+P+1,J+1)=0.0
C  CONCLUDES PARTIAL OF M
C  THE NEXT PART IS PARTIAL OF S
           PIPHI2=-4*A2*B2*(F+C*D*L)/L**5+A2*B2*(PF+C*D)/L**4
           P1S(I+1,J+1)=E*.25*H*T*T*IPHI2(I,J)
           P1S(I+P+1,J+P+1)=P1S(I+1,J+1)
           P2S(I+1,J+1)=E*.08333*T**3*IPHI2(I,J)
           P2S(I+1+P,J+1+P)=P2S(I+1,J+1)
           P3S(I+1,J+1)=E*.08333*H*T**3*PIPHI2
           P3S(I+1+P,J+1+P)=P3S(I+1,J+1)
140      CONTINUE
130      CONTINUE
C
      K=2*P+1
      DO 150 I=1,K
         P1S(1,I)=0.0
         P1S(I,1)=0.0
         P2S(1,I)=0.0
         P2S(I,1)=0.0
          P3S(1,I)=0.0
         P3S(I,1)=0.0
150       CONTINUE
C
      DO 160 I=1,P
         DO 170 J=1,P
           P1S(I+1,J+1+P)=0.0
           P1S(I+P+1,J+1)=0.0
           P2S(I+1,J+P+1)=0.0
           P2S(I+P+1,J+1)=0.0
           P3S(I+1,J+P+1)=0.0
           P3S(I+P+1,J+1)=0.0
170      CONTINUE
160      CONTINUE
C
      DO 180 I=1,P
```

```
      A=I*PI
      PBZ(I+1,2)=-(A/(L*L))*SIN(.5*A)
      PBZ(I+P+1,3)=-(A/(L*L))*SIN(A/2.0)
      PBZ(I+1,1)=0.0
      PBZ(I+1+P,1)=0.0
      PBZ(I+1,3)=0.0
      PBZ(I+1+P,2)=0.0
180   CONTINUE
    PBZ(1,1)=0.0
    PBZ(1,2)=0.0
    PBZ(1,3)=0.0
C
C  MULTIPLY PBZ *G1
    K=2*P+1
     CALL GAIN(G1,G2)
    CALL MATMUL(K,3,5,PBZ,G1,PBG1)
C  MULTIPLY PBG1*H
    CALL MATMUL(K,5,K,PBG1,HZ,PBG1H)
    CALL MATMUL(K,3,5,PBZ,G2,PBG2)
    CALL MATMUL(K,5,K,PBG2,HZ,PBG2H)
C  ASSEMBLE PARTIAL OF A AND PARTIAL OF B
    K=2*P+1
    DO 190 I=1,K
      DO 200 J=1,K
        KJ=J+K
        KI=I+K
        P1A(I,J)=0.0
        P2A(I,J)=0.0
        P3A(I,J)=0.0
        P1A(I,KJ)=-(P1M(I,J)*2)
        P2A(I,KJ)=-2*P2M(I,J)
        P3A(I,KJ)=-2*P3M(I,J)
        P1A(KI,J)=2*P1S(I,J)
        P2A(KI,J)=2*P2S(I,J)
        P3A(KI,J)=(2*P3S(I,J)+PBG1H(I,J))
        P1A(KI,KJ)=0.0
        P2A(KI,KJ)=0.0
        P3A(KI,KJ)=PBG2H(I,J)
200   CONTINUE
190   CONTINUE
C
    DO 210 I=1,K
      DO 220 J=1,K
        KI=I+K
        KJ=J+K
        P1B(I,J)=-2*P1M(I,J)
        P2B(I,J)=-2*P2M(I,J)
        P3B(I,J)=-2*P3M(I,J)
        P1B(I,KJ)=0.0
        P2B(I,KJ)=0.0
 P3B(I,KJ)=0.0
        P1B(KI,J)=0.0
        P2B(KI,J)=0.0
```

```
          P3B(KI,J)=0.0
          P1B(KI,KJ)=-2*P1M(I,J)
          P2B(KI,KJ)=-2*P2M(I,J)
          P3B(KI,KJ)=-2*P3M(I,J)
220     CONTINUE
210     CONTINUE
       CALL GRADNT(P1A,P2A,P3A,P1B,P2B,P3B,LIGVCT,
     1    RIGVCT,LBDA,PW3,PZ3)
        END

****************************************************************
****************************************************************

       SUBROUTINE GRADNT(P1A,P2A,P3A,P1B,P2B,P3B,LIGVCT,
     1   RIGVCT,LBDA,PW3,PZ3)
C  THIS PROGRAM CALCULATES THE GRADIENTS OF OMEGA
C    AND ZETA W.R.T THE PARAMETERS T, H , L
C
      REAL P1A(10,10),P2A(10,10),P3A(10,10),P1B(10,10),P2B(10,10)
      REAL P3B(10,10)
      COMPLEX LIGVCT(10,10),RIGVCT(10,10),LBDA(10),LTRANS(10,10)
      COMPLEX LPA1(10,10),LPA1R(10,10),LPB1(10,10),LPB1R(10,10)
      COMPLEX LPA2(10,10),LPA2R(10,10),LPB2(10,10),MAT(10,10)
      COMPLEX LPB2R(10,10),LPA3(10,10),LPA3R(10,10),LPB3(10,10)
      COMPLEX LPB3R(10,10),P1LBDA(10),P2LBDA(10),P3LBDA(10)
      INTEGER I,J,K,P,DIM
      REAL PW3(3),PZ3(3),DENOM,NUM1,NUM2
C

      P=2
      DIM=2*(2*P+1)
C
C  TRANSPOSE LIGVCT
      DO 20 I=1,DIM
        DO 30 J=1,DIM
          LTRANS(I,J)=LIGVCT(J,I)
30      CONTINUE
20      CONTINUE
C
C  CALCULATE THE PARTIAL DERIVATIVES OF LAMBDA
C
        DIM=10
C
C
      CALL MATCR(LTRANS,P1A,LPA1)
      CALL MTCPLX(LPA1,RIGVCT,LPA1R)
      CALL MATCR(LTRANS,P1B,LPB1)
      CALL MTCPLX(LPB1,RIGVCT,LPB1R)
C
C        PRINT*,'D LAMDA /DT  IS '
      DO 40 I=1,DIM
        DO 50 J=1,DIM
          MAT(I,J)=LPA1R(I,J)-LBDA(I)*LPB1R(I,J)
```

```
50        CONTINUE
           P1LBDA(I)=MAT(I,I)
C          PRINT*,P1LBDA(I)
40       CONTINUE
C
C    CALCULATE PARTIALS OF LAMBDA W.R.T. P2
         CALL MATCR(LTRANS,P2A,LPA2)
        CALL MTCPLX(LPA2,RIGVCT,LPA2R)
        CALL MATCR(LTRANS,P2B,LPB2)
        CALL MTCPLX(LPB2,RIGVCT,LPB2R)
C
C         PRINT*,'D LAMDA/DH  IS '
        DO 60 I=1,DIM
          DO 70 J=1,DIM
            MAT(I,J)=LPA2R(I,J)-LBDA(I)*LPB2R(I,J)
70        CONTINUE
           P2LBDA(I)=MAT(I,I)

C         PRINT*,P2LBDA(I)
60       CONTINUE
C
C   CALCULATE PARTIALS OF LAMBDA W.R.T. P3,  P3=L
C
        CALL MATCR(LTRANS,P3A,LPA3)
        CALL MTCPLX(LPA3,RIGVCT,LPA3R)
        CALL MATCR(LTRANS,P3B,LPB3)
        CALL MTCPLX(LPB3,RIGVCT,LPB3R)
C
C         PRINT*,'D LAMBDA /DL  IS '
        DO 80 I=1,DIM
          DO 90 J=1,DIM
            MAT(I,J)=LPA3R(I,J)-LBDA(I)*LPB3R(I,J)
90        CONTINUE
           P3LBDA(I)=MAT(I,I)
C         PRINT*,P3LBDA(I)
80       CONTINUE
C
C    FORM GRADIENTS OF W1 AND ZETA1
C   PW3(I) IS THE PARTIAL OF W(I) W.R.T. P1
C   PZ3(I) IS THE PARTIAL OF ZETA(1) W.R.T. P1
C
C
        DENOM=SQRT((REAL(LBDA(3)))**2+(AIMAG(LBDA(3)))**2)
C
          PW3(1)=(REAL(P1LBDA(3))+AIMAG(P1LBDA(3)))/DENOM
          PW3(2)=(REAL(P2LBDA(3))+AIMAG(P2LBDA(3)))/DENOM
          PW3(3)=(REAL(P3LBDA(3))+AIMAG(P3LBDA(3)))/DENOM
C
        NUM1=(REAL(LBDA(3)))**2
        NUM2=(REAL(LBDA(3)))*(AIMAG(LBDA(3)))
        PZ3(1)=-REAL(P1LBDA(3))/DENOM +(NUM1*(REAL(P1LBDA(3)))+NUM2
       1        *AIMAG(P1LBDA(3)))/DENOM**3
C
```

```
      PZ3(2)=-REAL(P2LBDA(3))/DENOM + (NUM1*(REAL(P2LBDA(3)))+NUM2
     1    *AIMAG(P2LBDA(3)))/DENOM**3
C
      PZ3(3)=(-REAL(P3LBDA(3))/DENOM) + (NUM1*(REAL(P3LBDA(3)))+
     1     NUM2*AIMAG(P3LBDA(3)))/DENOM**3
C
       END

      SUBROUTINE MATMUL(NR1,NR2,NCOL2,A,B,C)
       INTEGER NR1,NR2,NCOL2,I,J,K
       REAL SUM1,TEMP1
        REAL A(NR1,NR2),B(NR2,NCOL2),C(NR1,NCOL2)
        DO 10 I=1,NR1
          DO 20 J=1,NCOL2
        SUM1=0.0
            DO 30 K=1,NR2
          TEMP1=A(I,K)*B(K,J)
                 SUM1=SUM1+TEMP1
                 C(I,J)=SUM1
30       CONTINUE
20       CONTINUE
10       CONTINUE
        END
C

        SUBROUTINE MTCPLX(A,B,C)
C   THIS SUBROUTINE MULTIPLIES TWO COMPLEX MATRICES TOGETHER
C

       COMPLEX C(10,10)
        INTEGER NR1,NR2,NCOL2
          COMPLEX A(10,10),B(10,10)
          COMPLEX SUM1,TEMP1
      INTEGER I,J,K
C
        NR1=10
        NR2=10
        NCOL2=10
C  MULTIPLY GIVEN MATRICES
C
      DO 10 I=1,NR1
        DO 20 J=1,NCOL2
          SUM1 = 0.0

          DO 30 K=1,NR2
            TEMP1=A(I,K)*B(K,J)
            SUM1=SUM1+TEMP1
30       CONTINUE
         C(I,J)=SUM1
20      CONTINUE
10     CONTINUE
        END
C
```

```
C   THIS  SUBROUTINE  MULTIPLIES  A COMPLEX  AND  A  REAL  MATRIX
C     TOGETHER , THE MATRICES ARE 10 X 10
C
      SUBROUTINE MATCR(A,B,C)
      INTEGER NR1,NR2,NCOL2
       COMPLEX A(10,10),C(10,10)
       REAL B(10,10)
       COMPLEX SUM1,TEMP1
     INTEGER I,J,K
C
       NR1=10
       NR2=10
       NCOL2=10
C  MULTIPLY GIVEN MATRICES
C
      DO 10 I=1,NR1
        DO 20 J=1,NCOL2
          SUM1=0.0
          DO 30 K=1,NR2
            TEMP1=A(I,K)*B(K,J)
            SUM1=SUM1+TEMP1
30         CONTINUE
          C(I,J)=SUM1
20        CONTINUE
10         CONTINUE
      END

****************************************************************
****************************************************************
```

APPENDIX D

CONMIN OUTPUT FOR CASE 2

This appendix contains a more detailed listing of the output referred to in Chapter 4, Case 2. The listings provide the initial values for the decision variables, t, h, and L and the performance criteria used for the particular run, i.e. DELFUN, DABFUN, CT, and THETA. The final optimization information is also shown with the reason for termination and the number of iterations required for convergence or the number of iterations performed without convergence.

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                  C O N M I N                      *
*                                                   *
*              FORTRAN PROGRAM FOR                  *
*                                                   *
*       CONSTRAINED FUNCTION MINIMIZATION           *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *


CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT   NDV    ITMAX    NCON    NSIDE   ICNDIR   NSCAL   NFDG
  2       3      40       2        6       4        0       1

LINOBJ   ITRM    N1       N2       N3      N4       N5
  0       3       5       8        51      51       102

      CT              CTMIN             CTL             CTLMIN
 -.10000E+00       .40000E-02       -.10000E-01       .10000E-02

     THETA             PHI             DELFUN           DABFUN
  .10000E+01       .50000E+01       .10000E-03       .40507E-03

     FDCH             FDCHM            ALPHAX           ABOBJ1
  .10000E-01       .10000E-01       .10000E+00       .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
  1)      .10000E-05    .10000E-05    .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
  1)      .10000E+11    .10000E+11    .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR


INITIAL FUNCTION INFORMATION

OBJ =     .405072E+00

DECISION VARIABLES (X-VECTOR)
  1)      .97000E-02    .50000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .42565E+00   -.35929E-02
```

```
ITER =    1      OBJ =     .40507E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .97000E-02    .50000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)

1)      .42565E+00   -.35929E-02

ITER =    2      OBJ =     .40507E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .97000E-02    .50000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .42565E+00   -.35929E-02

ITER =    3      OBJ =     .40507E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .97000E-02    .50000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .42565E+00   -.35929E-02

ITER =    4      OBJ =     .40507E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .97000E-02    .50000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .42565E+00   -.35929E-02

ITER =    5      OBJ =     .41520E+00

DECISION VARIABLES (X-VECTOR)
  1)      .99422E-02    .50001E+00    .40001E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .28323E+00   -.23054E-02

ITER =    6      OBJ = .43479E+00


DECISION VARIABLES (X-VECTOR)
  1)      .10411E-01    .50002E+00    .40002E+01

CONSTRAINT VALUES (G-VECTOR)
```

```
  1)      .39871E-02  -.28098E-04

 ITER =     7      OBJ =     .43479E+00     NO CHANGE IN OBJ

 DECISION VARIABLES (X-VECTOR)
  1)      .10411E-01   .50002E+00   .40002E+01

 CONSTRAINT VALUES (G-VECTOR)
  1)      .39871E-02  -.28098E-04


 ITER =     8      OBJ =     .43479E+00     NO CHANGE IN OBJ

 DECISION VARIABLES (X-VECTOR)
  1)      .10411E-01   .50002E+00   .40002E+01

 CONSTRAINT VALUES (G-VECTOR)
  1)      .39871E-02  -.28098E-04


FINAL OPTIMIZATION INFORMATION

 OBJ =     .434785E+00

 DECISION VARIABLES (X-VECTOR)
  1)      .10411E-01   .50002E+00   .40002E+01

 CONSTRAINT VALUES (G-VECTOR)
  1)      .39871E-02  -.28098E-04

 THERE ARE     2 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     1    2

 THERE ARE 0 VIOLATED CONSTRAINTS

 THERE ARE     0 ACTIVE SIDE CONSTRAINTS

 TERMINATION CRITERION
      ABS(1-OBJ(I-1)/OBJ(I)) LESS THAN DELFUN FOR  3 ITERATIONS

 NUMBER OF ITERATIONS =     8

 OBJECTIVE FUNCTION WAS EVALUATED            24   TIMES

 CONSTRAINT FUNCTIONS WERE EVALUATED         24   TIMES

 GRADIENT OF OBJECTIVE WAS CALCULATED         5   TIMES

 GRADIENTS OF CONSTRAINTS WERE CALCULATED     5   TIMES
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                  C O N M I N                      *
*                                                   *
*              FORTRAN PROGRAM FOR                  *
*                                                   *
*       CONSTRAINED FUNCTION MINIMIZATION           *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *
```

CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT	NDV	ITMAX	NCON	NSIDE	ICNDIR	NSCAL	NFDG
2	3	40	2	6	4	0	1

LINOBJ	ITRM	N1	N2	N3	N4	N5
0	3	5	8	51	51	102

CT	CTMIN	CTL	CTLMIN
-.10000E+01	.40000E-02	-.10000E-01	.10000E-02
THETA	PHI	DELFUN	DABFUN
.20000E+01	.50000E+01	.10000E-03	.10000E-01
FDCH	FDCHM	ALPHAX	ABOBJ1
.10000E-01	.10000E-01	.10000E+00	.10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
1) .10000E-05 .10000E-05 .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
1) .10000E+11 .10000E+11 .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR

INITIAL FUNCTION INFORMATION

OBJ = .501120E+00

DECISION VARIABLES (X-VECTOR)
1) .15000E-01 .40000E+00 .40000E+01

CONSTRAINT VALUES (G-VECTOR)
1) -.23225E+01 .10955E-01

ITER = 1 OBJ = .50112E+00 NO CHANGE IN OBJ

```
DECISION VARIABLES (X-VECTOR)
  1)      .15000E-01    .40000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)

1)     -.23225E+01    .10955E-01


ITER =     2      OBJ =     .50112E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .15000E-01    .40000E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.23225E+01    .10955E-01


ITER =     3      OBJ =     .49979E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14959E-01    .40000E+00    .40002E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.22962E+01    .10871E-01


ITER =     4      OBJ =     .49846E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14919E-01    .40000E+00    .40004E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.22700E+01    .10787E-01


ITER =     5      OBJ =     .49714E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14879E-01    .40000E+00    .40006E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.22440E+01    .10702E-01


ITER =     6      OBJ =     .49582E+00

DECISION VARIABLES (X-VECTOR)
1)     .14838E-01    .40000E+00    .40008E+01
CONSTRAINT VALUES (G-VECTOR)
  1)     -.22180E+01    .10617E-01


ITER =     7      OBJ =     .49450E+00
```

```
DECISION VARIABLES (X-VECTOR)
  1)      .14798E-01    .40000E+00    .40010E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.21921E+01    .10532E-01


ITER =     8       OBJ =      .49319E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14758E-01    .40000E+00    .40012E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.21663E+01    .10446E-01


ITER =     9       OBJ =      .49188E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14718E-01    .40000E+00    .40014E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.21406E+01    .10360E-01


ITER =    10       OBJ =      .49188E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .14718E-01    .40000E+00    .40014E+01

CONSTRAINT VALUES (G-VECTOR)
  1)   -.21406E+01    .10360E-01



FINAL OPTIMIZATION INFORMATION

OBJ =      .491878E+00

DECISION VARIABLES (X-VECTOR)
  1)      .14718E-01    .40000E+00    .40014E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.21406E+01    .10360E-01

THERE ARE     0 ACTIVE CONSTRAINTS

THERE ARE     1 VIOLATED CONSTRAINTS
CONSTRAINT NUMBERS ARE
    2

THERE ARE     0 ACTIVE SIDE CONSTRAINTS
```

TERMINATION CRITERION
    TEN CONSECUTIVE ITERATIONS FAILED TO PRODUCE A FEASIBLE DESIGN

NUMBER OF ITERATIONS =   10

OBJECTIVE FUNCTION WAS EVALUATED	31	TIMES
CONSTRAINT FUNCTIONS WERE EVALUATED	31	TIMES
GRADIENT OF OBJECTIVE WAS CALCULATED	9	TIMES
GRADIENTS OF CONSTRAINTS WERE CALCULATED	9	TIMES

APPENDIX E

CONMIN OUTPUT FOR CASE 3

Case 3 elliminated the equality constraints of Case 2 and replaced them with inequality constraints. A region was specified to maintain the natural frequency of the third eigenvalue, in order to provide CONMIN more flexibility in the optimization process. Only a few of the many trials are listed to demonstrate that more than one local minimum exists due to the linearity of the objective function and the gradients of the eigenvalues with respect to the structural parameters.

```
 * * * * * * * * * * * * * * * * * * * * * * * * * * *
 *                                                   *
 *                   C O N M I N                     *
 *                                                   *
 *               FORTRAN PROGRAM FOR                 *
 *                                                   *
 *         CONSTRAINED FUNCTION MINIMIZATION         *
 *                                                   *
 * * * * * * * * * * * * * * * * * * * * * * * * * * *


     CONSTRAINED FUNCTION MINIMIZATION

     CONTROL PARAMETERS

     IPRINT   NDV    ITMAX    NCON    NSIDE   ICNDIR   NSCAL   NFDG
       2       3      40       2        6       4        0       1

     LINOBJ   ITRM    N1       N2      N3      N4       N5
       0       3      5        8       51      51       102

         CT                CTMIN             CTL              CTLMIN

 -.20000E+01       .40000E-02       -.10000E-01       .10000E-02

         THETA             PHI               DELFUN           DABFUN
       .50000E+01        .50000E+01        .10000E-03        .10000E+00

         FDCH              FDCHM             ALPHAX           ABOBJ1
       .10000E-01        .10000E-01        .10000E+00        .10000E+00

     LOWER BOUNDS ON DECISION VARIABLES (VLB)
       1)      .10000E-05   .10000E-05   .10000E-05

     UPPER BOUNDS ON DECISION VARIABLES (VUB)
       1)      .10000E+11   .10000E+11   .10000E+11

     ALL CONSTRAINTS ARE NON-LINEAR


     INITIAL FUNCTION INFORMATION

     OBJ =     .402372E+00

     DECISION VARIABLES (X-VECTOR)
       1)      .10417E-01   .50000E+00   .37000E+01

     CONSTRAINT VALUES (G-VECTOR)
       1)      .36013E+00  -.76013E+00
```

```
ITER =     1      OBJ =     .40237E+00      NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .10417E-01   .50000E+00   .37000E+01

CONSTRAINT VALUES (G-VECTOR)

1)       .36013E+00  -..76013E+00

ITER =     2      OBJ =     .40237E+00      NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .10417E-01   .50000E+00   .37000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .36013E+00  -.76013E+00

ITER =     3      OBJ =     .39224E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10491E-01   .48477E+00   .36938E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .35718E+00  -.75718E+00

ITER =     4      OBJ =     .19333E+00

DECISION VARIABLES (X-VECTOR)
  1)      .11930E-01   .21629E+00   .35882E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.26099E+00  -.13901E+00

ITER =     5      OBJ =     .19333E+00      NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .11930E-01   .21629E+00   .35882E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.26099E+00  -.13901E+00

ITER =     6      OBJ =     .18221E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12965E-01   .18786E+00   .35831E+01

CONSTRAINT VALUES (G-VECTOR)
```

```
  1)     -.91711E-04  -.39991E+00

 ITER =     7      OBJ =     .17126E+00

 DECISION VARIABLES (X-VECTOR)
   1)      .12186E-01    .18784E+00    .35831E+01

 CONSTRAINT VALUES (G-VECTOR)
   1)     -.40000E+00  -.28651E-06


FINAL OPTIMIZATION INFORMATION

 OBJ =      .171257E+00

 DECISION VARIABLES (X-VECTOR)
   1)      .12186E-01    .18784E+00    .35831E+01

 CONSTRAINT VALUES (G-VECTOR)
   1)     -.40000E+00  -.28651E-06

 THERE ARE     1 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     2

 THERE ARE     0 VIOLATED CONSTRAINTS

 THERE ARE     0 ACTIVE SIDE CONSTRAINTS

 TERMINATION CRITERION
      ABS(OBJ(I)-OBJ(I-1))    LESS THAN DABFUN FOR  3 ITERATIONS

 NUMBER OF ITERATIONS =     7

 OBJECTIVE FUNCTION WAS EVALUATED            19   TIMES

 CONSTRAINT FUNCTIONS WERE EVALUATED         19   TIMES

 GRADIENT OF OBJECTIVE WAS CALCULATED         5   TIMES

 GRADIENTS OF CONSTRAINTS W'.RE CALCULATED    5   TIMES
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                  C O N M I N                      *
*                                                   *
*              FORTRAN PROGRAM FOR                  *
*                                                   *
*      CONSTRAINED FUNCTION MINIMIZATION            *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *


CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT   NDV    ITMAX    NCON    NSIDE   ICNDIR   NSCAL   NFDG
  2       3      40       2        6        4       0       1

LINOBJ   ITRM    N1       N2      N3       N4      N5
  0       3       5       8       51       51      102

     CT              CTMIN             CTL              CTLMIN
 -.10000E+00      .40000E-02      -.10000E-01        .10000E-02

    THETA             PHI             DELFUN            DABFUN
  .10000E+01      .50000E+01       .10000E-03        .43430E-03

    FDCH             FDCHM            ALPHAX            ABOBJ1
  .10000E-01      .10000E-01       .10000E+00        .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
  1)      .10000E-05   .10000E-05   .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
  1)      .10000E+11   .10000E+11   .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR


INITIAL FUNCTION INFORMATION

OBJ =     .434304E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10400E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.22037E+00  -.17963E+00


ITER =     1      OBJ =     .42158E+00
```

```
DECISION VARIABLES (X-VECTOR)
  1)      .10095E-01    .49999E+00    .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.40260E+00    .26001E-02


ITER =     2       OBJ =     .38001E+00

DECISION VARIABLES (X-VECTOR)
  1)      .11363E-01    .40405E+00    .39638E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.73960E-03   -.39926E+00


ITER =     3       OBJ =     .35673E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10668E-01    .40403E+00    .39638E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.40023E+00    .22547E-03


ITER =     4       OBJ =     .31854E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12125E-01    .31952E+00    .39380E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.67936E-03   -.39932E+00


ITER =     5       OBJ =     .31851E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12127E-01    .31943E+00    .39380E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.13530E-04   -.39999E+00


ITER =     6       OBJ =     .31851E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12127E-01    .31943E+00    .39380E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.62211E-07   -.40000E+00


ITER =     7       OBJ =     .31835E+00
```

```
DECISION VARIABLES (X-VECTOR)
  1)      .12138E-01    .31899E+00    .39379E+01

CONSTRAINT VALUES (G-VECTOR)
  1)      .32826E-02  -.40328E+00


FINAL OPTIMIZATION INFORMATION

 OBJ =     .318350E+00

 DECISION VARIABLES (X-VECTOR)
   1)      .12138E-01    .31899E+00    .39379E+01

 CONSTRAINT VALUES (G-VECTOR)
   1)      .32826E-02  -.40328E+00

 THERE ARE     1 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     1

 THERE ARE     0 VIOLATED CONSTRAINTS

 THERE ARE     0 ACTIVE SIDE CONSTRAINTS

 TERMINATION CRITERION
      ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  3 ITERATIONS

 NUMBER OF ITERATIONS =     7

 OBJECTIVE FUNCTION WAS EVALUATED             22   TIMES

 CONSTRAINT FUNCTIONS WERE EVALUATED          22   TIMES

 GRADIENT OF OBJECTIVE WAS CALCULATED          7   TIMES

 GRADIENTS OF CONSTRAINTS WERE CALCULATED      7   TIMES
```

APPENDIX F

CONMIN OUTPUT FOR CASE 4

In Case 4 the frequency for the third eigenvalue was constrained to remain between 4.2 and 5.2 and the first eigenvalue was constrained to the left half plane. In Case 3 the objective funciton was minimized and the frequency of the third eigenvalue maintained in a specified region, but the first eigenvalue was unconstrained and hence moved to an unstable region. Since this instability can occur, the first eigenvalue was constrained to the left half plane. Three of the many trials attempted are listed here as referenced in Case 4.

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                    C O N M I N                    *
*                                                   *
*                FORTRAN PROGRAM FOR                *
*                                                   *
*         CONSTRAINED FUNCTION MINIMIZATION         *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *


CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT   NDV     ITMAX    NCON     NSIDE   ICNDIR   NSCAL    NFDG
  2       3       40       3         6       4        0        1

LINOBJ   ITRM     N1       N2        N3      N4       N5
  0       3        5        8        51      51      102

     CT              CTMIN             CTL             CTLMIN
 -.10000E+01      .40000E-02      -.10000E-01      .10000E-02

    THETA             PHI            DELFUN            DABFUN
  .30000E+01      .50000E+01      .10000E-03      .43500E-03

     FDCH            FDCHM           ALPHAX            ABOBJ1
  .10000E-01      .10000E-01      .10000E+00      .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
  1)      .10000E-05   .10000E-05   .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
  1)      .10000E+11   .10000E+11   .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR


INITIAL FUNCTION INFORMATION

OBJ =     .434997E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10417E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.81038E+00  -.18962E+00  -.80229E-01

** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET
```

```
ITER =     1      OBJ =     .43500E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .10417E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.81038E+00  -.18962E+00  -.80229E-01


ITER =     2      OBJ =     .43340E+00
DECISION VARIABLES (X-VECTOR)
  1)      .10378E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.83335E+00  -.16665E+00  -.80799E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     3      OBJ =     .42943E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10283E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.89042E+00  -.10958E+00  -.77555E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     4      OBJ =     .42755E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10238E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.91730E+00  -.82699E-01  -.33336E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     5      OBJ =     .42737E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10234E-01   .50000E+00   .40000E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.91989E+00  -.80109E-01  -.44576E-02
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     6      OBJ =     .42037E+00
```

```
DECISION VARIABLES (X-VECTOR)
  1)      .10440E-01    .48287E+00    .39936E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.85074E+00   -.14926E+00   -.81351E-01
** CONSTRAINT     3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     7      OBJ =     .41684E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10352E-01    .48287E+00    .39936E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.90293E+00   -.97074E-01   -.21008E-01
** CONSTRAINT     3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     8      OBJ =     .41684E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .10352E-01    .48287E+00    .39936E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.90293E+00   -.97074E-01   -.21008E-01


ITER =     9      OBJ =     .41674E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10350E-01    .48287E+00    .39936E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.90438E+00   -.95625E-01   -.20532E-02
** CONSTRAINT     3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =    10      OBJ =     .41673E+00

DECISION VARIABLES (X-VECTOR)
  1)      .10350E-01    .48287E+00    .39936E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.90452E+00   -.95484E-01   -.67406E-05


FINAL OPTIMIZATION INFORMATION

OBJ =     .416732E+00
```

```
        DECISION VARIABLES (X-VECTOR)
          1)      .10350E-01    .48287E+00    .39936E+01

        CONSTRAINT VALUES (G-VECTOR)
          1)     -.90452E+00   -.95484E-01   -.67406E-05

        THERE ARE     1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
            3

        THERE ARE     0 VIOLATED CONSTRAINTS

        THERE ARE     0 ACTIVE SIDE CONSTRAINTS

        TERMINATION CRITERION
             ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  3 ITERATIONS

        NUMBER OF ITERATIONS =    10

        OBJECTIVE FUNCTION WAS EVALUATED            28   TIMES

        CONSTRAINT FUNCTIONS WERE EVALUATED         28   TIMES

        GRADIENT OF OBJECTIVE WAS CALCULATED         8   TIMES

        GRADIENTS OF CONSTRAINTS WERE CALCULATED     8   TIMES
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                   C O N M I N                     *
*                                                   *
*               FORTRAN PROGRAM FOR                 *
*                                                   *
*        CONSTRAINED FUNCTION MINIMIZATION          *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *
```

CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT	NDV	ITMAX	NCON	NSIDE	ICNDIR	NSCAL	NFDG
2	3	40	3	6	4	0	1

LINOBJ	ITRM	N1	N2	N3	N4	N5
0	3	5	8	51	51	102

CT	CTMIN	CTL	CTLMIN
-.10000E+00	.40000E-02	-.10000E-01	.10000E-02

THETA	PHI	DELFUN	DABFUN
.30000E+01	.50000E+01	.10000E-03	.28184E-03

FDCH	FDCHM	ALPHAX	ABOBJ1
.10000E-01	.10000E-01	.10000E+00	.10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
1) .10000E-05 .10000E-05 .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
1) .10000E+11 .10000E+11 .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR

INITIAL FUNCTION INFORMATION

OBJ = .281844E+00

DECISION VARIABLES (X-VECTOR)
1) .12750E-01 .26953E+00 .39279E+01

CONSTRAINT VALUES (G-VECTOR)
1) -.59846E+00 -.40154E+00 -.89384E-01

** CONSTRAINT 3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET

```
ITER =    1     OBJ =    .28085E+00

DECISION VARIABLES (X-VECTOR)
  1)     .12705E-01   .26953E+00   .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.62202E+00  -.37798E+00  -.90451E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET

ITER =    2     OBJ =    .27999E+00

DECISION VARIABLES (X-VECTOR)
  1)     .12666E-01   .26953E+00   .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.64234E+00  -.35766E+00  -.91158E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET

ITER =    3     OBJ =    .27764E+00

DECISION VARIABLES (X-VECTOR)
  1)     .12560E-01   .26952E+00   .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.69757E+00  -.30243E+00  -.80412E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET

ITER =    4     OBJ =    .27735E+00

DECISION VARIABLES (X-VECTOR)
  1)     .12547E-01   .26952E+00   .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.70438E+00  -.29562E+00   -.55992E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET

ITER =    5     OBJ =    .27719E+00

DECISION VARIABLES (X-VECTOR)
  1)     .12540E-01   .26952E+00   .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.70820E+00  -.29180E+00  -.11972E-02
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET
```

```
ITER =     6      OBJ =     .27719E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12540E-01    .26952E+00    .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.70827E+00   -.29173E+00   -.11783E-04
** CONSTRAINT     3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     7      OBJ =     .27719E+00

DECISION VARIABLES (X-VECTOR)
  1)      .12540E-01    .26952E+00    .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.70827E+00   -.29173E+00   -.16167E-06


FINAL OPTIMIZATION INFORMATION

OBJ =     .277187E+00


DECISION VARIABLES (X-VECTOR)
  1)      .12540E-01    .26952E+00    .39279E+01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.70827E+00   -.29173E+00   -.16167E-06
THERE ARE     1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
    3

THERE ARE     0 VIOLATED CONSTRAINTS

THERE ARE     0 ACTIVE SIDE CONSTRAINTS

TERMINATION CRITERION
     ABS(OBJ(I)-OBJ(I-1))    LESS THAN DABFUN FOR  3 ITERATIONS

NUMBER OF ITERATIONS =     7

OBJECTIVE FUNCTION WAS EVALUATED            22   TIMES

CONSTRAINT FUNCTIONS WERE EVALUATED         22   TIMES

GRADIENT OF OBJECTIVE WAS CALCULATED         7   TIMES

GRADIENTS OF CONSTRAINTS WERE CALCULATED     7   TIMES
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                     *
*                    C O N M I N                      *
*                                                     *
*                FORTRAN PROGRAM FOR                  *
*                                                     *
*         CONSTRAINED FUNCTION MINIMIZATION           *
*                                                     *
* * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

```
CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT   NDV    ITMAX    NCON    NSIDE   ICNDIR   NSCAL   NFDG
  2       3      40        3       6        4       0       1

LINOBJ   ITRM    N1       N2      N3      N4      N5
  0       3       5        8      51      51     102

       CT              CTMIN             CTL             CTLMIN
  -.10000E+01       .40000E-02      -.10000E-01       .10000E-02

     THETA              PHI             DELFUN           DABFUN
   .30000E+01       .50000E+01       .10000E-03       .18545E-03

      FDCH             FDCHM            ALPHAX           ABOBJ1
   .10000E-01       .10000E-01       .10000E+00       .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
   1)      .10000E-05    .10000E-05    .10000E-05

UPPER BOUNDS ON DECISION VARIABLES (VUB)
   1)      .10000E+11    .10000E+11    .10000E+11

ALL CONSTRAINTS ARE NON-LINEAR


INITIAL FUNCTION INFORMATION

OBJ =      .185449E+00

DECISION VARIABLES (X-VECTOR)
   1)      .15300E-01    .15000E+00    .38700E+01

CONSTRAINT VALUES (G-VECTOR)
   1)    -.50768E+00   -.49232E+00   -.99915E-01
```

```
ITER =     1      OBJ =     .17108E+00

DECISION VARIABLES (X-VECTOR)
  1)      .15647E-01   .13650E+00   .38361E+01

CONSTRAINT VALUES (G-VECTOR)
  1)  -.49714E+00  -.50286E+00  -.55735E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     2      OBJ =     .17108E+00     NO CHANGE IN OBJ
DECISION VARIABLES (X-VECTOR)
  1)      .15647E-01   .13650E+00   .38361E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.49714E+00  -.50286E+00  -.55735E-01


ITER =     3      OBJ =     .17108E+00     NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .15647E-01   .13650E+00   .38361E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.49714E+00  -.50286E+00  -.55735E-01
** CONSTRAINT    3 HAS ZERO GRADIENT
DELETED FROM ACTIVE SET


ITER =     4      OBJ =     .17102E+00

DECISION VARIABLES (X-VECTOR)
  1)      .15642E-01   .13650E+00   .38361E+01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.49962E+00  -.50038E+00   .10340E-02


FINAL OPTIMIZATION INFORMATION

 OBJ =     .171015E+00

 DECISION VARIABLES (X-VECTOR)
  1)      .15642E-01   .13650E+00   .38361E+01

 CONSTRAINT VALUES (G-VECTOR)
  1)     -.49962E+00  -.50038E+00   .10340E-02

 THERE ARE     1 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     3
```

```
THERE ARE     0 VIOLATED CONSTRAINTS

THERE ARE     0 ACTIVE SIDE CONSTRAINTS

TERMINATION CRITERION
     ABS(OBJ(I)-OBJ(I-1))    LESS THAN DABFUN FOR  3 ITERATIONS

NUMBER OF ITERATIONS =     4

OBJECTIVE FUNCTION WAS EVALUATED               10   TIMES

CONSTRAINT FUNCTIONS WERE EVALUATED            10   TIMES

GRADIENT OF OBJECTIVE WAS CALCULATED            3   TIMES

GRADIENTS OF CONSTRAINTS WERE CALCULATED        3   TIMES
```

THIS DATA REPRESENTS A REDUCTION IN THE MASS OF 60.686%

THESE ARE THE EIGENVALUES FOR THE RECONFIGURED MODEL

EIGENVALUE 1 IS:

-.89657897E-02
-.40263574E+01

AND ITS RIGHT EIGENVECTOR IS:

.20283846E-12	-.20713803E-12
.20287406E-03	-.12120727E-01
.75890674E-04	-.42990834E-02
-.20287408E-03	.12120727E-01
-.75890681E-04	.42990834E-02
-.17978025E-10	-.13965871E-10
-.10000000E+01	.17264924E-08
-.35469399E+00	-.32444053E-03
.10000000E+01	.00000000E+00
.35469399E+00	.32444113E-03

ITS LEFT EIGENVECTOR IS :

-.99686658E-01	.14922314E+00
.10000000E+01	.24836869E-12
.31033788E+00	.88445692E-01
.10000000E+01	-.24590960E-14
.31033788E+00	.88445692E-01
-.39535199E-01	-.15906706E-01
.63175826E-01	.20093084E-01
-.63906318E-03	-.36791595E-04
.63175826E-01	.20093084E-01
-.63906318E-03	-.36791595E-04

EIGENVALUE 2 IS:

-.89657897E-02
.40263574E+01

AND ITS RIGHT EIGENVECTOR IS:

.20283846E-12	.20713803E-12
.20287406E-03	.12120727E-01
.75890674E-04	.42990834E-02
-.20287408E-03	-.12120727E-01
-.75890681E-04	-.42990834E-02
-.17978025E-10	.13965871E-10
-.10000000E+01	-.17264924E-08
-.35469399E+00	.32444053E-03
.10000000E+01	.00000000E+00
.35469399E+00	-.32444113E-03

ITS LEFT EIGENVECTOR IS :

-.99686658E-01	-.14922314E+00
.10000000E+01	-.248348869E-12
.31033788E+00	-.884[illegible]692E-01
.10000000E+01	.24590960E-14
.31033788E+00	-.88445692E-01
-.39535199E-01	.15906706E-01
.63175826E-01	-.20093084E-01
-.63906318E-03	.36791595E-04
.63175826E-01	-.20093084E-01
-.63906318E-03	.36791595E-04

EIGENVALUE 3 IS:

-.15201891E+00
.46979208E+01

AND ITS RIGHT EIGENVECTOR IS:

-.33983587E-11	.11201286E-11
.68806857E-02	.21263747E+00
.52025281E-03	-.38222131E-02
-.68806857E-02	-.21263747E+00
-.52025281E-03	.38222131E-02
-.50343469E-11	-.15770762E-10
-.10000000E+01	-.37357219E-11
.17877366E-01	.30251551E-02
.10000000E+01	-.298380355E-14
-.17877366E-01	-.30251551E-02

ITS LEFT EIGENVECTOR IS :

-.25502852E-02	-.62046723E-02
.10000000E+01	.16462116E-14
.33511027E+00	-.76985047E-01
-.94879122E+00	.55318496E-02
-.31910987E+00	.72996150E-01
-.13559820E-02	.38728660E-03
.442267740E-01	-.10299691E-01
-.82829267E-03	-.62820201E-04
-.40681695E-01	.93731696E-02
.78781128E-03	.623329975E-04

EIGENVALUE 4 IS:

-.15201891E+00
-.46979208E+01

AND ITS RIGHT EIGENVECTOR IS:

-.33983587E-11	-.11201286E-11
.68806857E-02	-.21263747E+00

.52025281E-03	.38222131E-02
-.68806857E-02	.21263747E+00
-.52025281E-03	-.38222131E-02
-.50343469E-11	.15770762E-10
-.10000000E+01	.37357219E-11
.17877366E-01	-.30251551E-02
.10000000E+01	.29838035E-14
-.17877366E-01	.30251551E-02

ITS LEFT EIGENVECTOR IS :

-.25502852E-02	.62046723E-02
.10000000E+01	-.16462116E-14
.33511027E+00	.76985047E-01
-.94879122E+00	-.55318496E-02
-.31910987E+00	-.72996150E-01
-.13559820E-02	-.38728660E-03
.44226740E-01	.10299691E-01
-.82829267E-03	.62820201E-04
-.40681695E-01	-.93731696E-02
.78781128E-03	-.62332975E-04

EIGENVALUE 5 IS:

-.25908072E+00
.40835030E+01

AND ITS RIGHT EIGENVECTOR IS:

-.55304663E-03	-.24836221E+00
-.80832378E-04	.11485410E+00
.29670804E-05	.46093571E-02
.12712803E-03	.11589202E+00
.18784851E-05	.46031533E-02
.10000000E+01	-.17816479E-14
-.46244291E+00	-.13552177E-02
-.18558945E-01	-.29379992E-04
-.46662383E+00	-.52720058E-03
-.18533957E-01	-.33707454E-04

ITS LEFT EIGENVECTOR IS :

-.76659446E-01	-.16735915E+00
.10000000E+01	-.28147774E-12
.31654856E+00	-.94705402E-01
.10000000E+01	-.30266423E-14
.31654856E+00	-.94705402E-01
-.42808413E-01	.12489566E-01
.65476509E-01	-.19552827E-01
-.67137883E-03	-.82860496E-05
.65476509E-01	-.19552827E-01
-.67137883E-03	-.82860496E-05

EIGENVALUE 6 IS:

```
-.25908072E+00
-.40835030E+01
```

AND ITS RIGHT EIGENVECTOR IS:

-.55304663E-03	.24836221E+00
-.80832378E-04	-.11485410E+00
.29670804E-05	-.46093571E-02
.12712803E-03	-.11589202E+00
.18784851E-05	-.46031533E-02
.10000000E+01	.17816479E-14
-.46244291E+00	.13552177E-02
-.18558945E-01	.29379992E-04
-.46662383E+00	.52720058E-03
-.18533957E-01	.33707454E-04

ITS LEFT EIGENVECTOR IS :

-.76659446E-01	.167335915E+00
.10000000E+01	.28147774E-12
.31654856E+00	.94705402E-01
.10000000E+01	.30266423E-14
.31654856E+00	.94705402E-01
-.42808413E-01	-.12489566E-01
.654765O9E-01	.19552827E-01
-.67137883E-03	.82860496E-05
.654765O9E-01	.19552827E-01
-.67137883E-03	.82860496E-05

EIGENVALUE 7 IS:

```
-.12842362E+01
 .83071612E+02
```

AND ITS RIGHT EIGENVECTOR IS:

-.58702105E-05	.20529406E-03
-.18605265E-03	-.12034930E-01
-.70882689E-04	-.42177970E-02
-.18643520E-03	-.12032994E-01
-.71017753E-04	-.42171103E-02
-.17046570E-01	-.75129393E-03
.10000000E+01	-.82216874E-15
.35047023E+00	-.47169158E-03
.99983963E+00	-.34265667E-04
.35041336E+00	-.48379340E-03

ITS LEFT EIGENVECTOR IS :

-.15658675E-01	-.17234424E-05
.10000000E+01	-.75480239E-10
.35089629E+00	-.98417340E-06
.10000000E+01	.15314432E-14
.35089629E+00	-.98414663E-06
-.11804776E-04	.18835837E-03
.19973735E-03	-.12032557E-01
.65130251E-04	-.42222112E-02
.19973735E-03	-.12032557E-01
.65130251E-04	-.42222112E-02

EIGENVALUE 8 IS:

-.12842362E+01
-.83071612E+02

AND ITS RIGHT EIGENVECTOR IS:

-.58702105E-05	-.20529406E-03
-.18605265E-03	.12034930E-01
-.70882689E-04	.42177970E-02
-.18643520E-03	.12032994E-01
-.71017753E-04	.42171103E-02
-.17046570E-01	.75129393E-03
.10000000E+01	.82216874E-15
.35047023E+00	.47169158E-03
.99983963E+00	.34265667E-04
.35041336E+00	.48379340E-03

ITS LEFT EIGENVECTOR IS :

-.15658675E-01	.17234424E-05
.10000000E+01	.75480239E-10
.35089629E+00	.98417340E-06
.10000000E+01	-.15314432E-14
.35089629E+00	.98414663E-06
-.11804776E-04	-.18835837E-03
.19973735E-03	.12032557E-01
.65130251E-04	.42222112E-02
.19973735E-03	.12032557E-01
.65130251E-04	.42222112E-02

EIGENVALUE 9 IS:

-.13805354E+01
-.82480192E+02

AND ITS RIGHT EIGENVECTOR IS:

-.15474786E-01	.24390597E+00
-.33921607E-02	-.11598615E+00
.10534196E-03	-.46137411E-02
-.34871759E-02	-.11711928E+00
.11074735E-03	-.46065659E-02
.10000000E+01	-.18226307E-14
-.47275093E+00	.43901672E-01
-.18867518E-01	.76516716E-03
-.47735349E+00	.44583241E-01
-.18839618E-01	.74123524E-03

ITS LEFT EIGENVECTOR IS :

-.30398316E-06	.27938072E-06
-.99983410E+00	-.36113721E-04
-.35463975E+00	-.13311750E-04
.10000000E+01	.00000000E+00
.35469847E+00	.63802115E-06
-.45390183E-08	-.34669883E-08
-.20807377E-03	-.12117204E-01
-.71675598E-04	-.42979627E-02
.20854182E-03	.12119207E-01
.71838934E-04	.42986721E-02

EIGENVALUE 10 IS:

-.13805354E+01
.82480192E+02

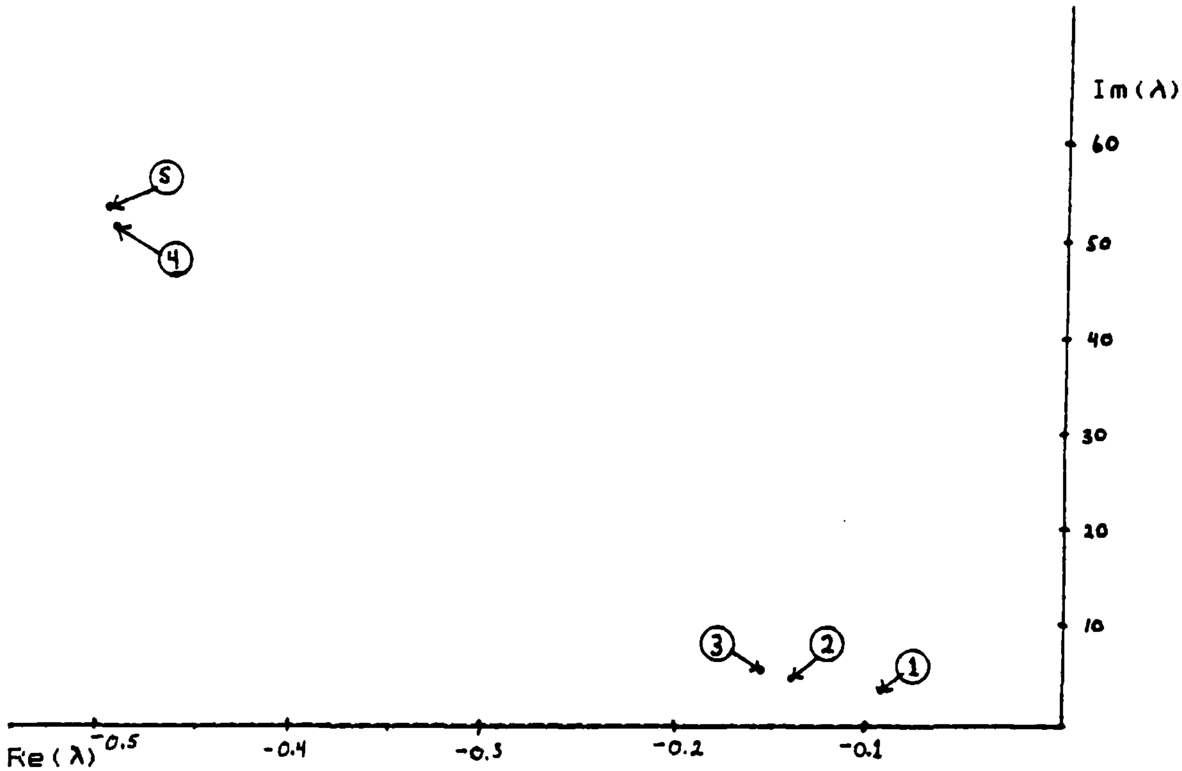
AND ITS RIGHT EIGENVECTOR IS:

-.15474786E-01	-.24390597E+00
-.33921607E-02	.11598615E+00
.10534196E-03	.46137411E-02
-.34871759E-02	.11711928E+00
.11074735E-03	.46065659E-02
.10000000E+01	.18226307E-14
-.47275093E+00	-.43901672E-01
-.18867518E-01	-.76516716E-03
-.47735349E+00	-.44583241E-01
-.18839618E-01	-.74123524E-03

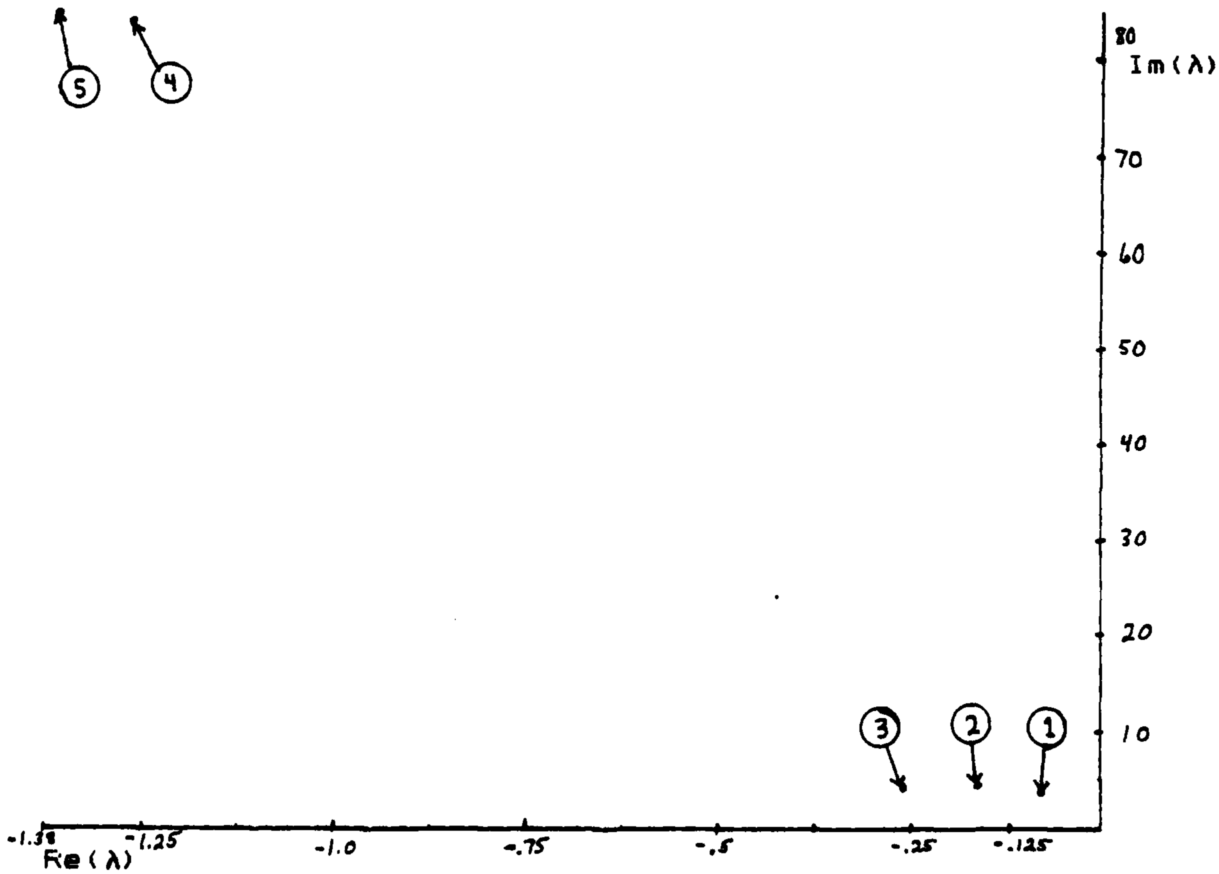
ITS LEFT EIGENVECTOR IS :

-.30398316E-06	-.27938072E-06
-.99983410E+00	.36113721E-04
-.35463975E+00	.13311750E-04
.10000000E+01	.00000000E+00
.35469847E+00	-.63802115E-06
-.45390183E-08	.34669883E-08
-.20807377E-03	.12117204E-01

-.71675598E-04	.42979627E-02
.20854182E-03	-.12119207E-01
.71838934E-04	-.42986721E-02



Root Locus of First Five Modes of Initial Design



Root Locus of First Five Modes of Revised Design

VITA

Thomas V. Muckenthaler was born in Tulsa, Oklahoma on May 9, 1954. Soon after graduation from East Central High School in Tulsa, he began his Air Force career as a cadet at the USAF Academy in Colorado Springs, Colo. Graduating in 1976 with a Bachelor of Science in Engineering Mechanics, Lieutenant Muckenthaler went on to USAF Undergraduate Pilot Training at Vance AFB, Okla. which he successfully completed in August 1977. He was then assigned to the 916th Air Refueling Squadron, Travis AFB, Calif where he upgraded to Aircraft Commander in the KC-135A. A Masters Degree in Systems Management from the University of Southern California was obtained during his stay at Travis AFB. In May 1983 Captain Muckenthaler was reassigned to the School of Engineering, Air Force Institute of Technology, to enter the Masters Degree Program in Astronautical Engineering. Upon graduation he will be assigned to Headquarters Space Command in Colorado Springs, Colo.

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74128

SECURITY CLASSIFICATION OF THIS PAGE

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ABSTRACT

An eigenspace optimization approach is used to incorporate optimal control into the structural design process for large flexible space structures. The equations of motion for an uncontrolled system are developed by deriving the kinetic and potential energy for the system and then using assumed modes to discretize the energies. These expressions are then linearized, the Lagrangian formed, and Lagrange's equations written for the system. An existing optimal control law is incorporated to form the equations of motion for the controlled system. A parameter optimization technique is used to minimize the mass of the Draper/RPL Configuration model involving eigenspace optimization. A computer algorithm is developed that effectively optimizes a global structural parameter vector to minimize the mass of the model, while constraining specified eigenvalues. The eigenvalue sensitivities are passed to a constrained function minimization program called CONMIN which minimizes the mass of the appendages. The constraints imposed restrict the first eigenvalue to the left half plane and the natural frequency of the third eigenvalue to a specified stable region. The result is an algorithm that incorporates an existing optimal control law into the structural optimization process.

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